Designing the Public Service Systems with an Exact Optimization Core

Jan Bendik*
Department of Mathematical Methods and Operations Research
Faculty of Management Science and Informatics
University of Žilina
Univerzitná 8215/1, 010 26 Žilina, Slovakia
Jan.Bendik@fri.uniza.sk

Abstract
This paper deals with the problem of designing the public service systems with an exact optimization core. Designing the public service system represents NP-hard problem consisting of solution to the p-median location problem. Erlenkotter designed one of the most effective algorithm for solving the uncapacitated facility location problem. Erlenkotter approach is based on the branch and bound method, theory of duality and using dual solution to obtaining the lower and upper bound of solution. we present two approaches to solving the p-median location problem with using Erlenkotter approach. Semi-exact iterative approach is based on the transformation of the p-median location problem to the uncapacitated facility location problem by Lagrangean relaxation of the p-median condition. Generalized exact approach is based on the generalization of Erlenkotter approach to the solving the p-median location problem. The proposed approaches are compared in terms of demands on the computational time and accuracy of the obtained solution.

Categories and Subject Descriptors
G.1.6 [**Optimization**]: Integer Programming

Keywords
Public service system design, location problem, Erlenkotter approach, Lagrangean relaxation, branch and bound method, generalized exact approach, semi-exact iterative approach, compositional approach.

*Recommended by thesis supervisor: Prof. Jaroslav Janáček
Defended at Faculty of Management Science and Informatics, University of Žilina on August 22, 2016.

Bendik, J. Designing the Public Service Systems with an Exact Optimization Core. Information Sciences and Technologies Bulletin of the ACM Slovakia, Vol. 9, No. 1 (2017) 9-15

1. Introduction
In real situations, we find various service systems, which have become part of our lives. Service system can be divided into private and public service systems.The servise system design consists of a set of customers and a set of candidates to the facility location. Private service systems is based on the maximal profit of system owner. Main goal of private servise system is minimizing costs to requirements of the customers with some profit. The public service system structure is formed by the deployment of limited number of service centers and the associated objective is to minimize social costs, which are often proportional to distances from serviced customers to the nearest source of provided service.The service is available for all customers. Designing a public service system, including medical emergency system, fire-brigade deployment, public administration system and many others, can often bring along some overall combinatorial problems concerning the system structure. The public system design can evaluate the quality solutions using different criteria. The system criterium gives minimizing the sum of the costs associated with the establishment of centers and customer servicing. Minimax criterion gives position of the worst locations of customer to the nearest center. Criterion of solidarity takes into account fairness of customer service.The public service system design is NP hard problem [6],[7] and often related to the uncapacitated facility location problem or p-median problem. This problem is formulated as a task of determination of at most p network nodes as facility locations. The number of possible service center locations seriously impacts the computational time. To obtain good decision on facility location in any serviced area, a mathematical model of the problem can be formulated and some of mathematical programming methods can be applied to find the optimal solution. Real instances of the problem are characterized by big numbers of possible service center locations, which can take the value of several hundreds or thousands. Many authors have deals with this problem. Balinski [2] provided an early integer programming formulation of the plant location problem which has historically been adapted to the p-median problem. Reese [10] summarized the exact solution methods for the p-median problem. Avella, Sassano and Vasilev [1] designed a branch-and-price-and-cut algorithm to solve large-scale instances of the p-median problem. Garcia, Labbe, Marin [5] designed an effective Z-Erlange and BRanch Algoritmus for solving the p-median problem based on the covering approach. Erlenkotter [4] used knowledges from theory of duality and

proposed one of the most effective algorithms $DualLoc$ for solving the uncapacitated facility location problem. Erlenkotter designed approach is based on the branch and bound method with specific approach to the calculation the lower and upper bound. Inspired by this approach Korkel [9] improved the Erlenkotter approach and designed the algorithm $PDLoc$. Janacek and Buzna [8] designed algorithm $BBDual$ for solving the uncapacitated facility location problem based on the knowledges from the Erlenkotter and Korkel approach. The paper deals with methods of designing the public service systems using informatics resources . The public service system design is based on the solving p-median location problem. We used knowledges from the Erlenkotter approach and designed two approaches to solving the p-median location problem with using Erlenkotter approach. Semi-exact iterative approach is based on the transformation of the p-median location problem to the uncapacitated facility location problem by Lagrangean relaxation of the p-median condition. Generalized exact approach is based on the generalization of Erlenkotter approach to the solving the p-median location problem. The proposed approaches with various modification are compared in terms of demands on the computational time and accuracy of the obtained solution.

2. Problem of the Public Service System Design

The p-median location problem finds the optimal location of maximal p facilities and the sum of the distances between the closest facilities and their costumers is minimized. The p-median location problem consists of placing facilities such as hospitals, police stations in some sites of a given finite set I. The placing facilities serve the customers such as villages and cities from a given finite set J. The total costs of the optimal deployment of facilities in the specific network are constituted the fixed charges f_i and the costs c_{ij}. The fixed charges f_i give costs for the facility location at the location i. The costs c_{ij} give costs for the demand satisfaction of the j-th customer from the location i.
The p-median location problem can be modeled using the following notation. Let the decision of the service center location at the place $i \in I$ be modeled by a zero-one variable $y_i \in \{0,1\}$ which takes the value of 1, if a center is located at i, otherwise it takes the value of 0. In addition, the variables $z_{ij} \in \{0,1\}$ for each $i \in I$ and $j \in J$ represent to assign a customer j to a possible location i by the value of 1. The maximal number of the facility locations represents the constant p. The mathematical model for the p-median location problem is defined as follows:

$$Minimize \quad F_P = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} \quad (1)$$

$$Subject\ to: \quad \sum_{i\in I} z_{ij} = 1 \quad \forall j \in J \quad (2)$$

$$z_{ij} \le y_i \quad \forall i \in I, \forall j \in J \quad (3)$$

$$\sum_{i\in I} y_i \le p \quad (4)$$

$$y_i \in \{0,1\} \quad \forall i \in I \quad (5)$$

$$z_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (6)$$

The objective function (1) represents the minimization of the total costs of the p-median location problem which consists of the fixed charges f_i and the costs c_{ij}. The constraints (2) ensure that each customer is assigned to the exactly one possible service center location. Binding constraints (3) enable to assign a customer to a possible location i, only if the service center is located at this location. The constraint (4) bounds the maximal number of the located service centers. The obligatory conditions in the mathematical model are (5) and (6). The defined mathematical model of p-median location problem gives the combination of the uncapacitated facility location problem and the p-median problem.

3. Iterative Approach with Lagrangean Relaxation

The designed iterative approach with Lagrangean relaxation provides us the possibility of solving the p-median problem. This approach is based on the transformation of the p-median location problem to the uncapacitated facility location problem.

3.1 Mathematical Model of Relaxation Problem

The mathematical model (1)-(6) using the Lagrangean relaxation of the constraint (4) which limits number of located centers is modified as follows:

$$Minimize \quad F_R = \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} z_{ij} + \\ + Lg(\sum_{i\in I} y_i - p) \quad (7)$$

$$St: \quad \sum_{i\in I} z_{ij} = 1 \quad \forall j \in J \quad (8)$$

$$z_{ij} \le y_i \quad \forall i \in I, \forall j \in J \quad (9)$$

$$y_i \in \{0,1\} \quad \forall i \in I \quad (10)$$

$$z_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (11)$$

We repeat the solution of the model (7)-(11) with a change of the Lagrange multiplier Lg for obtaining the optimal solution of the mathematical model (1)-(6) until the last member of the objective function (7) is equal to zero. If the last member of the objective function (7) is non-equal to zero and the setting the Lagrangean multiplier gives the most suitable value then we do not obtain optimal solution, but only near-to-optimal solution. The value of the optimal solution is located somewhere between the value of non-relaxation problem solving (1) and value of the LP-relaxation solving (7).

3.2 Iterative Algorithm with Improving Heuristic

The iterative approach with Lagrangean relaxation is realized by algorithm $pMBBDual$. The solution of the mathematical model (7)-(11) represents one iteration of the algorithm $pMBBDual$. The value of objective function (7) of solving problem (7)-(11) provides the lower bound of the problem (1)-(6) obtained by algorithm $BBDual$. The algorithm $BBDual$ represent the exact optimization core of algorithm $pMBBDual$, where the optimal solution of algorithm $BBDual$ with some value of Lagrangean multiplier Lg corresponds with a solution in one iteration of the algorithm $pMBBDual$.The quality and the feasibility of the solution of the suggested approach depends on a suitable setting of the Lagrangean multiplier Lg. The

suitable value of the Lagrangean multiplier is obtained by a bisection algorithm. We repeat the solution of the model (7)-(11) with a change of the Lagrange multiplier Lg for obtaining the optimal solution of the mathematical model (1)-(6) until the last member of the objective function (7) is equal to zero. If the last member of the objective function (7) is non-equal to zero and the setting the Lagrangean multiplier gives the most suitable value then we do not obtain optimal solution, but only a near-to-optimal solution. The value of the optimal solution is located somewhere between the value of non-relaxation problem solving (1) and value of the LP-relaxation solving (7). Algorithm $pMBBDual$ which realized designed approach does not provide the optimal solution to the p-median location problem all the time. If algorithm does not obtain the optimal solution then we improve a near-to-optimal solution by some heuristic. We designed 3 heuristics to the improving the solution obtained by algorithm $pMBBDual$. The designed heuristics work on the assumption that we have the best feasible or infeasible solution or both solutions. The best feasible solution is the solution where number of locations is smaller and the nearest to p locations. The solution obtained by the algorithm $pMBBDual$ is the best feasible solution. The best infeasible solution is the solution where number of locations is bigger and the nearest to p locations. We verified designed heuristics on the benchmark from slovak road network to the computational time and quality of solution. Based on the executed experiments an insertion heuristic with the best admissible strategy was the most suitable to the improving the solution obtained by algorithm $pMBBDual$.
An insertion heuristic with the best admissible strategy works on the assumption that we have the best feasible solution. This obtained feasible solution is accompanied by the not included locations, which bring the greatest improvement. If number of locations the obtained feasible solution is r and $r < p$, then we find the not included locations with the greatest improvement for the solution with number of locations $r + 1$. If we find the locations with the greatest improvement, then include the found location to the solution. This process of finding and including locations repeats for obtaining the best solution with number of locations $r + 2, r + 3, .., p$. The insertion heuristic ends when the number of locations is equal p. The number of accompanied locations is equal the difference (p - r).

3.3 Iterative Algorithm with Estimation of Lagrangean Multiplier

The quality and the feasibility of the solution obtained by algorithm $pMBBDual$ of the suggested approach depends on a suitable setting of the Lagrangean multiplier Lg. The suitable value of the Lagrangean multiplier is obtained by a bisection algorithm. Obtaining the most suitable setting of Lagrangean multiplier can be time-consuming, when we execute a lot of iterations. We improved algorithm $pMBBDual$ to the setting of Lagrangean multiplier. We designed alternative method to the bisection method based on the estimation of fixed charges of service system. We analysed the dependence the number of facility location to the setting the fixed charges. We tried to estimate fixed charges f for the best feasible solution with number of facility location p and decrease a number of the executed iteration of algorithm $pMBBDual$. A Value of fixed charges is equal the most suitable setting of Lagrangean multiplier .Let m gives the cardinality of set I, p_F gives the number of facilty location with fixed charges F, then we can formulate a estimation of the Lagrange multipliers as follows:

$$f = F\left(\frac{(m-p)p_F}{(m-p_F)p}\right)^{\alpha} \qquad (12)$$

We tried to obtain value of exponent α, which can provided the best estimation of Lagrangean multiplier. Based on the executed experiments the best setting of exponent α represents valuu 1.1. desigbed approach repairs bad estimation with r-section with coefficient of interval division r is equal to 8.

3.4 Comparison of the Iterative Algorithms

Designed itterative approaches to the solving the public service system design was compared on the benchmarks from Slovak road network in the computational time and the quality of the obtained solutions. We tested the p-median problem and the weighted p-median problem. We evaluated the executed experiments based on created statistics (see Table 1). Statistics pt gives average computational time in tested benchmark to solved problem in seconds, statistic ppI gives the average number of iterations and statistic pU gives number of interuption after one hour of running the algorithm. Column *Bench* in the Table 1 gives tested benchmark.
Basic algorithm represents algorithm based on setting the Lagrangean relaxation with bisection algorithm. Obtained solution is improved with an insertion heuristic with the best admissible strategy.
Improved algorithm represents algorithm based on setting the Lagrangean relaxation with estimation of Lagrangean multiplier. Obtained solution is improved with an insertion heuristic with the best admissible strategy.

Evaluation of experiments based on defined statistics (see Table 1) shows that that the obtaining of Lagrange multipliers with estimation is better in average computational time and a smaller number of executed iterations compared with bisection algorithms. A large number of iterations affects the large computational time, where time restriction algorithm may cause premature stopping.

Table 1: Comparison of Basic and Improved Semi-exact Iterative Algorithm pMBBDDual

	Basic			Improved		
Bench	**pt[s]**	**ppI**	**ppU**	**pt[s]**	**ppI**	**ppU**
BA_p	0,07	13,1	0	0,05	7,4	0
BB_p	38,99	13,7	0	11,49	5,9	0
KE_p	188,63	13,6	0	42,97	7,5	0
PO_p	696,23	13,6	1	383,15	7,5	0
NR_p	42,12	13,5	0	6,6	7,4	0
TN_p	0,58	13,6	0	0,22	7,2	0
TT_p	5,63	13,6	0	2,03	7,4	0
ZA_p	1,83	13,7	0	0,49	6,2	0
SR_p	3600,0	2,0	58	3600,0	2,0	58
BA_wp	0,02	10	0	0,02	8,3	0
BB_wp	0,96	12,6	0	0,23	6	0
KE_wp	0,48	12,2	0	0,21	8,1	0
PO_wp	4,43	13,4	0	1,17	7,8	0
NR_wp	0,31	12,8	0	0,16	8,4	0
TN_wp	0,13	12,6	0	0,08	7,8	0
TT_wp	0,11	12,3	0	0,09	8,9	0
ZA_wp	0,13	12,1	0	0,07	6,8	0
SR_wp	635,47	13,5	0	189,98	6,3	0

4. Generalized Erlenkotter Approach

Generalized Erlenkotter approach represents to solving the problem of public service system design (1)-(6) than p-median location problem and designed the exact algorithm $pMedBBDual$.
Erlenkotter approach represents solving the uncapacitated facility location problem using by the theory of duality and the branch and bound method which designed Erlenkotter [4] in 1976. The basic idea of Erlenkotter approach consists in relation between linear relaxation of the original problem and the associated dual problem. Inspired by Erlenkotter approach we used also the theory of duality and generalized the Erlenkotter approach to the solving p-median location problem. After some reformulation and introduction of slack variables u_i, the dual problem of p-median location problem can be formulated as follows (12-16):

$$Maximize \qquad F_D = \sum_{j \in J} v_j + px \tag{13}$$

$$St: \sum_{j \in J} max(0, v_j - c_{ij}) + x + u_i = f_i \qquad \forall i \in I \tag{14}$$

$$v_j \geq 0 \qquad \forall j \in J \tag{15}$$

$$u_i \geq 0 \qquad \forall i \in I \tag{16}$$

$$x \leq 0 \tag{17}$$

In dual model variable v_j corresponds with condition (2) and dual variable x corresponds with condition (4). Slack Variables u_i provide equality of conditions (14).
Based on knowledges from Erlenkotter approach we designed the exact algorithm $pMedBBdual$ to the solving p-median location problem. Algorithm $pMedBBDual$ realizes the branch and bound method. The important parameters in the branch and bound method are computation of lower bound, computation of upper bound of branch, choice of a candidate to the branching and searching strategy.
We designed Correction Dual Ascent algorithm (CDA) for obtaining the lower bound of solution. CDA algorithm solves the dual problem. Obtaining solution of dual model(13-17) provide a lower bound for optimal solution of the problem (1)-(6). The Correction Dual Ascent procedure (CDA) starts from an arbitrary feasible solution (18)-(20) of the dual problem and subsequently increases the values of the v_j variables and decreases the value of the x variable as long as complementary conditions hold.

$$v_j = min(c_{ij:i \in I}) \qquad \forall j \in J \tag{18}$$

$$u_i = f_i \qquad \forall i \in I \tag{19}$$

$$x = 0 \tag{20}$$

Algorithm CDA used the DA algorithm which designed Janacek and Buzna [8].We tried to improve value of the lower bound with Dual Adjustment algorithm (DA). A lower bound for optimal solution of the problem (1-6) constitutes objective function value of arbitrary feasible solution of (13-17). Generalized Erlenkotter approach solves the dual problem (13-17) and uses the obtained dual solution to the obtaining the induced feasible solution applying the complementary conditions (21-24).

$$(y_i - z_{ij}) max(0, v_j - c_{ij}) = 0 \qquad \forall i \in I, \forall j \in J \tag{21}$$

$$u_i y_i = 0 \qquad \forall i \in I \tag{22}$$

$$max(0, c_{ij} - v_j) z_{ij} = 0 \qquad \forall i \in I, \forall j \in J \tag{23}$$

$$(p - \sum_{i \in I} y_i) x = 0 \tag{24}$$

According to the complementary slackness theorem a primal feasible solution and a dual feasible solution are both optimal only if the respect all complementary conditions. F_D represents a value of objective function of dual problem (12-16). F_P represents a value of objective function of the p-median location problem (1-6). Differences between the values of objective functions of the primal and dual solution gives sum of left sides all complementary conditions (21-24):

$$F_P - F_D = \sum_{i \in I} \sum_{j \in J} (y_i - z_{ij}) max(0, v_j - c_{ij}) + \\ + \sum_{i \in I} u_i y_i + \sum_{i \in I} \sum_{j \in J} max(0, c_{ij} - v_j) z_{ij} + \\ + (\sum_{i \in I} y_i - p) x \tag{25}$$

Computation of upper bound in designed algorithm ensures the function for obtaining the minimal set of locations. Minimal set of locations gives primal solution and is based in effort to respect of complementary conditions in the greatest extent. This function ensures always respect of condition (24) and tries to obtain the minimal set of locations with the minimal gap (25). Function for obtaining the minimal set location with Erlenkotter order of customers selects the location where u_i is equal to zero, $v_j \geq c_{ij}$ and minimalize the conditions (24). It is not possible to ensure always the satisfaction of conditions (21)-(24) when we construct of a minimal set. The created gap with function for obtaining the minimal set of locations is possible to decrease with the good choice of the candidate to the branching. Choice of a candidate ensures function for obtaining the candidate (fractional variable) to the branching which choose a fractional variable to the 0-1 fixation from the minimal set of locations in the branch and bound method. The function for obtaining the candidate is based on the evaluation of the complementary conditions (21)-(24) and finding the first location from the set of locations which does not satisfy the conditions (21)-(24). Respect of condition (24) ensures function for minimal set of locations. Algorithm $pMedBBDual$ evaluates respect of the complementary conditions in order (21-22-23). Searching strategy represents choice of node to the branching which can be processed like first. Algorithm $pMedBBDual$ realizes the depth first search but with choice of branch according to smaller lower bound than searching strategy. Exception in the searching strategy is priority processing of the node from pair of nodes which will be exclude from the branching. We tested algorithm pMedBBDual on the Slovak road network in computational time. It shows that which is shown that the obtaining of the optimal solution is time consuming.

4.1 Improving the Exact Optimization Core

Improving the exact optimization core represents research benefits or limits of the designed method for obtaining a candidate to the branching, the lower and upper bound

in the designed algorithm $pMedBBDual$.
We tried to analyze what is the reason of the long time with $pMedBBDual$ algorithm which realizes a generalized approach in an effort to improve the proposed algorithm. We used the value of optimal solution obtained by universal IP solver $XPRESS - IVE$ than upper bound of algorithm solution. This setting of upper bound shows us the quality of lower bound of algorithm.Experiments showed that algorithm CDA does not provide good lower bound. So we modified CDA algorithm and designed Increment Correction Dual Ascent algorithm $ICDA$. $ICDA$ algorithm is not fixed to some customer than CDA algorithm. Decreasing the dual variable x in $ICDA$ is step by step about one. $ICDA$ algorithm provide better lower bound to the quality than CDA algorithm.
We identified that processing of nodes with initial values of dual variables in the branch and bound method is better. We have dealt order of condition complementary evaluation and studied used function for obtaining candidate to the fixation. we designed new variants of function with difference order of condition evaluation. Experiments, realized statistics and statistic test ((t-Test: Two-Sample Assuming Unequal Variances) showed that evaluation of condition (22) like first can bring benefits. We choose order (22)-(21)-(23) than the best order of condition evaluation. We modified the function for selecting the minimal set of locations. This selection is fixed to the order of processing the individual customers. We designed few variants of function for selection of minimal set with the different order of processing the customers. Experiments showed feasibility of used function with Erlenkotter order of customers.

4.2 Comparison of Algorithms with the Generalized Erlenkotter Approach

Designed exact generalized provide optimal solution, but somethimes in long computational time. So we limited the computatuonal time to the one hour. Designed exact approaches to the solving the public service system design was compared on the benchmarks from Slovak road network in the computational time and the quality of the obtained solutions. We tested the p-median problem and the weighted p-median problem. We evaluated the executed experiments based on created statistics (see Table 2).

Statistic OR gives number of the obtained optimal solution to the number of the executed experiments in tested benchmark to the solved problem in percentage. Statistics pt gives average computational time in tested benchark to solved problem in seconds and statistic pU gives number of interuption after one hour of running the algorithm. In the Table 2 column Bench gives the tested benchmark. Symbol - gives that we do not know the optimal solution so we cannot evaluate statistics.
Basic algorithm represents algorithm consists of CDA method for obtaining the lower bound, function for obtaining the candidate to the fixation with evaluation of condition in order (21)-(22)-(23), function for minimal set of location with Erlenkotter order of processed costumers.
Improved algorithm represents algorithm consists of $ICDA$ method, function for obtaining the candidate to the fixation with evaluation of condition in order (22)-(21)-(23), function for minimal set of location with Erlenkotter order of processed costumers.
The evaluation experiments on the defined statistics (see Table 2) showed that the improved algorithm was better in all measured statistics. The exception was the one tested by region where the basic algorithm was better in the average time to the solving p-median problem, as reflected less premature stopping algorithm, but improved algorithm was bettr in obtaining optimal solutions. In other cases, the improved algorithm was better in all defined statistics even if it the algorithm was interrupt.

Table 2: Comparison of Exact Algorithms of Generalized Erlenkotter Approach

	BASIC			IMPROVED		
Bench	**OR**	**pt[s]**	**pU**	**OR**	**pt[s]**	**pU**
BA_p	80	1153,33	5	100	0,15	0
BB_p	80	2097,54	6	84	898,30	5
KE_p	87	1565,26	5	87	1804,65	11
PO_p	73	3301,86	28	85	1241,65	10
NR_p	74	1270,80	6	96	577,17	3
TN_p	78	1040,56	5	100	4,84	0
TT_p	94	597,15	2	100	361,59	1
ZA_p	50	2083,02	11	100	125,43	0
SR_p	-	3600,00	58	-	3487,59	56
BA_wp	94	868,05	4	100	0,39	0
BB_wp	60	2505,25	12	100	207,92	0
KE_wp	39	2989,27	17	100	147,70	0
PO_wp	27	3380,25	29	97	762,86	3
NR_wp	43	2279,40	14	100	134,37	0
TN_wp	56	2226,30	11	100	9,15	0
TT_wp	75	939,89	4	100	6,98	0
ZA_wp	50	2221,46	12	100	13,43	0
SR_wp	2	3600,00	58	0	3600,00	58

5. Comparison of Semi-exact Iterative Algorithm and Generalized Exact Algorithm

Designed improved approaches to the solving the public service system design was compared on the Beasley benchmarks [3] and benchmarks from Slovak road network in the computational time and the quality of the obtained solutions. We tested the uncapacitated facility location problem, the p-median problem and the weighted p-median problem. We evaluated the executed experiments based on created statistics (see Table 3). Statistic OS gives number of the obtained optimal solution to the number of the executed experiments in tested benchmark to the solved problem in percentage. Statistic *Gap* gives the average value of relative gap between the values of the obtained solution and the optimal solution in percentage. Statistics *pt* gives average computational time in tested benchmark to solved problem in seconds. Column *Bench* in the Table 3 gives the tested benchmark. Symbol - gives that we do not know optimal solution so we cannot evaluate statistics.
Improved algorithm pMBBDual represents algorithm based on setting the Lagrangean relaxation with estimation of Lagrangean multiplier. Obtained solution is improved with an insertion heuristic with the best admissible strategy.
Improved algorithm pMedBBDual represents algorithm consists of $ICDA$ method to the obtaing the lower bound, function for obtaining the candidate to the fixation with evaluation of condition in order (22)-(21)-(23), function for minimal set of location with Erlenkotter order of processed costumers.
Statistics in the Table 3 showed that algorithm *pMBBDual* with improving heuristic provides much better computational times in the comparison with algorithm

Table 3: Comparison of Semi-exact Iterative Algorithm and Generalized Exact Algorithm

	pMBBDual			pMedBBDual		
Bench	**OS**	**Gap**	**pt[s]**	**OS**	**Gap**	**pt[s]**
BA_p	93	0,03	0,05	100	0,00	0,15
BB_p	72	0,04	11,49	84	0,01	898,30
KE_p	78	0,09	42,97	87	0,05	1804,65
PO_p	76	0,07	383,15	85	0,03	1241,65
NR_p	83	0,04	6,60	96	0,00	577,17
TN_p	83	0,04	0,22	100	0,00	4,84
TT_p	81	0,07	2,03	100	0,00	361,59
ZA_p	75	0,10	0,49	100	0,00	125,43
SR_p	-	-	3600,0	-	-	3487,6
BA_wp	100	0,00	0,02	100	0,00	0,39
BB_wp	100	0,00	0,23	100	0,00	207,92
KE_wp	100	0,00	0,21	100	0,00	147,70
PO_wp	97	0,00	1,17	97	0,00	762,86
NR_wp	100	0,00	0,16	100	0,00	134,37
TN_wp	100	0,00	0,08	100	0,00	9,15
TT_wp	100	0,00	0,09	100	0,00	6,98
ZA_wp	95	0,00	0,07	100	0,00	13,43
SR_wp	93	0,00	189,98	0	17,90	3600,00
Beasley	100	0,00	0,74	86	0,00	1450,46
SR1000	100	0,00	10,45	0	86	0,00

Table 4: Comparison of Improving Algorithm with Compositional Approach Inquality of Soluton and Computatinal Time

	VIA		VZA		CA	
Bench	**GR**	**pt[s]**	**GR**	**pt[s]**	GR	pt[s]
BA_p	73	0,05	100	0,15	100	0,16
BB_p	68	11,49	80	898,3	84	567,18
KE_p	70	42,97	52	1804,65	83	803,13
PO_p	70	383,15	70	1241,65	94	659,77
NR_p	74	6,60	87	577,17	100	10,65
TN_p	72	0,22	100	4,84	100	1,1
TT_p	69	2,03	94	361,59	100	23,54
ZA_p	60	0,49	100	125,43	100	4,1
SR_p	-	3600	3	3488	-	-
BA_wp	100	0,02	100	0,39	100	0,02
BB_wp	100	0,23	100	207,92	100	0,23
KE_wp	100	0,21	100	147,7	100	0,21
PO_wp	97	1,17	97	762,86	100	3,73
NR_wp	100	0,16	100	134,37	100	0,16
TN_wp	100	0,08	100	9,15	100	0,08
TT_wp	100	0,09	100	6,98	100	0,09
ZA_wp	95	0,07	100	13,43	100	0,23
SR_wp	93	189,98	0	3600,0	93	423,61
Beasley	100	0,74	100	4,51	100	0,74
SR1000	100	10,45	71	1450,5	100	10,45

pMedBBDual. Algorithm pMedBBDual provides better statistics OS and Gap in the comparison with algorithm pMBBDual with the improving heuristic in most of the tested cases. It means that algorithm pMedBBDual provides optimal solution in most of tested cases, but with longer time. The computational times for solving p-median problem on the large-scale benchmarks can be extreme longer with the both algorithms. Algorithm pMBBDual was better in all statsistics for the uncapacitated facility location problem ($UFLP$) and we obtained optimal solutions to the weighted p-median problem on the whole slovak road network in most of tested cases. If algorithm pMedBBDual ends after the one hour of runnnig time then we obtain only the the near-to-optimal solution. Based on the all statistics we prefer the iterative approach with the improving heuristic.

6. Compositional Approach

Compositional approach is based on the compromise between quality of obtained solution and computational time and consists of the two phase. First phase represents obtaining near-to-solution with semi-exact iterative approach with improving heuristic. Second phase represents improvement of the obtained solution with the generalized exact approach using the lower and upper bound of solution from first phase. Designed compositional approach to the solving the public service system design was compared with improved algorithms $pMedBBDual$ and $pMBBDual$ on the Beasley benchmarks [3] and benchmarks from Slovak road network in the computational time and the quality of the obtained solutions. We tested the uncapacitated facility location problem, the p-median problem and the weighted p-median problem. We evaluate the executed experiments based on created statistics (see Table 4). Statistic GR gives number of the obtained optimal solution (only if lower bound is equal to upper bound of solution) to the number of the executed experiments in tested bencharK to the solved problem in percentage. Statistics *pt* gives average computational time in tested bencharK to solved problem in seconds. In the Table 4 column Bench gives the tested benchmark. Columns VIA represent iterative approach improved by with improving heuristic and columns VZA represent generalized Erlenkotterov approach.Columns CA represent compositional approach.

The results of numerical experiments (see Table 4) showed using a compositional approach to solving the public service system design.We obtained the optimal solution on average in less computational time than generalized algorithm pMedBBDual. Compositional approach was able to obtain an optimal solution in a larger number of tested cases than with the designed algorithms $pMBBDual$ with improving heuristic or algorithms $pMedBBDual$ in reducated computational time to one hour.The optimal solution was obtained an exact semi-iterative algorithm in 86% of all experiments, generalized algorithm in 77% of all experiments and compositional algorithms in 97% of all experiments in reducated computational time to one hour and obtaining always some feasible solutions (ie. Without p- median on the Slovakia). A limit of commpositional approach and algorithm pMBBDual represents the p-median problem on the whole slovak road network, when we cannot obtain some good feasible solution with the algorithm pMBBDual even after one hour calculation. Compositional algorithm can provide optimal solution in most cases in a relatively short computational time to the solved problem, which I consider to be the benefits of this work.

7. Summary

A public service system structure is formed by deployment of a limited number of the service centers and the associated objective is to minimize total costs. Designing a public service system, including medical emergency system, fire-brigade deployment, public administration system and many others, can often bring along some overall combinatorial problems concerning the system structure. The public service system design is NP hard problem and often related to the uncapacitated facility location problem or p-median problem. This problem is formulated as a task of determination of at most p network nodes as facility locations. The number of possible service center locations seriously impacts the computational time. To obtain good decision on facility location in any serviced area, a mathematical model of the problem can be formulated and some of mathematical programming methods can be applied to find the optimal solution. Real instances of the problem are characterized by big numbers of possible service center locations, which can take the value of several hundreds or thousands. The optimal solution of solved problem can be obtained by the universal IP solvers only for smaller instances of the problem. The universal IP solvers are very time consuming and often fail when solving a large instance. Many authors have deals with this problem. Erlenkotter designed one of the most effective algorithm for solving the uncapacitated facility location problem. Erlenkotter approach is based on the branch and bound method, theory of duality and using dual solution to obtaining the lower and upper bound of solution. The public service system design represents solving the p-median location problem which is combination of the p-median problem and the uncapacitated facility location problem. We designed two approaches for solving the public service system design. First approach is based on the Erlenkotter approach and the Lagrangean relaxation of the constraint which limits number of the located center. Lagrangean relaxation provides transformation of p-median location problem to the uncapacitated facility location problem. The quality of the resulting solutions depends on the convenient setting of the Lagrangean multiplier which is obtained by a bisection algorithm. We designed the better alternative to the most convenient setting of the Lagrangean multiplier based on the forecast of Lagrangean multiplier. Iterative semi-exact approach provide near-to-optimal solution and lower bound of solution, so we improved near-to-optimal solution with an insertion heuristic with the best admissible strategy. Second approach is based on the generalization of the Erlenkotter approach and branch and bound method. We generalized the Erlenkotter dual approach to the lower and upper bounding to be able to solve the associated location problem with the restricted number of the located service centers. We improved designed approach using the most appropriate combination of methods for obtaining candidates to the branching, the lower and upper bound. We compared designed approaches to the solving the public services system design in quality of the obtained solution and the computational times on the Beasley benchmarks and benchmarks from the Slovak road network. Iterative semi-exact approach provides only near-to-optimal solution, but in short computational time. Generalized exact approach provides optimal solution, but sometimes with extreme long time. We designed the compositional approach based on the two phase. First phase represents obtaining near-to-solution with semi-exact iterative approach with improving heuristic. Second phase represents improvement of the obtained solution with the generalized exact approach using the lower and upper bound of solution from first phase. Designed compositional approach gives the possibility of using the Erlenkotter approach to the solving the public service system design, which we consider for main benefit my thesis.

Acknowledgements. This work was supported by the research grants VEGA 1/0518/15 Resilient rescue systems with uncertain accessibility of service, VEGA 1/0463/16 - Economically effcient charging infrastructure deployment for electric vehicles in smart cities and communities and APVV-0760-11 Designing of Fair Service Systems on Transportation Networks.We would also like to thank to Centre of excellence for systems and services of intelligent transport (ITMS 26220120050) for built up the infrastructure, which was used.This work was partially supported by the Agency of the Slovak Ministry of Education for the Structural Funds of the EU, under project ITMS:26220120007 Centre of excellence in Computer Science and Knowledge Systems.

References

[1] Avella, P., Sassano, A., Vassil'ev, I.: *Computational study of large scale p-median problems*, Mathematical Programming 109 pp. 89-114, (2007)

[2] Balinski, M.: *Integer programming, methods, uses and computation*, Management Science. 12(3), pp. 254-313 (1965)

[3] Beasley J. E.: *OR Library: Distributing Test Problems by Electronic Mail*, Journal of the Operational Research Society, 41(11), pp 1069-1072, (1990)

[4] Erlenkotter, D.: *A Dual-Based procedure for uncapacitated facility location. Operations Research*, 26(6), pp. 992-1009 (1978)

[5] García, S., Labbe, M., Marin, A.: *Solving large p-median problems with a radius formulation*, INFORMS Journal on Computing 23 (4) pp. 546-556, (2011)

[6] Garey, M. R., Johnson, D. S.: *Computers and intractability: A guide to the theory of NP-completeness. A Series of Books in the Mathematical Sciences*, San Francisco, USA: W.H. Freeman and Company (1979)

[7] Hakimi, S. L., Kariv, C.: *An algorithmic approach to network location problems*, SIAM Journal on AppliedMathematics, 37(3), 539–560 (1979)

[8] Janacek, J., Buzna, L.: *An acceleration of Erlenkotter-Korkelt's algorithms for the uncapacitated facility location problem*, Annals of Operations Research, 164(1), pp. 97-109 (2008)

[9] Korkel, M.: *On the exact solution of large ? scale simple plant location problem*, European Journal of Operational Research, 39(2), pp. 157-173 (1898)

[10] Reese, J.: Solution methods for the p-median problem. Networks, 48(3), 125-?142 (2006)

Selected Papers by the Author

J. Bendík. *Exact algorithm for Location Problem Solving in Public Service System Design* In: Úlohy diskrétní optimalizace v dopravní praxi 2013 : SW nástroje na řešení úloh racionalizace. Objektivizace a optimalizace pokrytí území veřejnými obslužnými systémy, Pardubice, Česká republika, pp. 7-15.

J. Bendík. *Generalization of the Erlenkotter approach for solving of the public service system design*, In. Proc. of ICTTE 2014: Int. Conf. on Traffic and Transport Engineering: Belgrade, Serbia, 2014, pp. 12-17.

J. Bendík. *Heuristics for improving the solution of p-median location problem with Erlenkotter approach* In Proc. of the 10th int. conf. Digital technologies: Žilina, Slovakia, 2014, pp. 7-11.

J. Bendík. *Increment approach for obtaining the lower bound in public service system design* In Proc. of the 4th int. symp. and 26th national conf. on operational research: Chania-Greece, 2015, pp. 230-234.

J. Bendík. *Improving the exact algorithm for solving the public service system design in the branching* In SOR ť15 : Proc. of the 13th Int. Symp. on Operational Research: Bled, Slovenia, 2015, pp. 215-220.

J. Bendík. *Selection of minimal set of locations in the public service system design* In Proc. of Informatics 2015 : IEEE 13th int. scientific conf. on informatics: Poprad, Slovakia, 2015, pp. 47-51.

J. Bendík. *Solving the p-median location problem with the Erlenkotter approach in public service system design* In 4th Student Conf. on Operational Research: Nottingham, UK, 2014, Saarbrücken/Wadern: Dagstuhl Publishing, pp. 25-33.