Time Series Forecasting with Hybrid Neural Networks and Advanced Statistical Methods

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Abstract

The disseration thesis deals with modelling and forecasting of financial time series with hybrid statistical-soft-computing model of feedforward artificial neural networks due to improving forecasting qualities of standard models. The reason for this is that standard models do not have satisfied forecasting qualities in forecasting time series which are highfrequentional, very dynamic and very volatile. The main goal of this thesis was to construct and implement a new hybrid model of ANN with better prediction qualities than standard models. We created the new teoretical Error-Correction model of the neural network with autocorrection mechanism as well as its computational version RBF-SEMA. We suggested also other hybrid models in this work, i.e. RBF-KM, RBF-GA, ARCH-RBF, R-ARCH-RBF, W-ARCH-RBF, RBF-KM-SEMA, RBF-GA-SEMA. Except for mathematical proof the ECM model as well as other models were validated empirically on real financial data of exchange rates. The conlusion from experiments is that ECM model of ANN as well as RBF-KM and RBF-GA have significantly better prediction qualities than standard model and have a big potential in financial forecasting. Besides the new implemented hybrid models, the main contribution of this thesis is also in the new theory of ECM for neural networks.

Categories and Subject Descriptors

C.1.3. [**Computer Systems Organization**]: OTHER ARCHITECTURE STYLES—*Neural Nets*; I.2.6. [**Computing Methodologies**]: ARTIFICIAL INTELLIGENCE—*Learning - Connectionism and Neural Nets*; F.1.1. [**Theory of Computation**]: COMPUTATION BY ABSTRACT DEVICES—*Models of Computations - Self Modifying Machines*; I.2.8.. [**Computing Methodologies**]: ARTIFICIAL INTELLIGENCE—*Problem Solving, Control Methods, and Search*

Keywords

error-correction, ECM, neural network, radial basis function, genetic algorithm, financial time series, hybrid models, forecasting

1. Introduction

Nowadays, machine learning is using in almost every aspect of our lives, financial area not excluding. Today, computers are present also in this sphere; the execution business has transferred from stock-market spots into the virtual world. Modern finance uses techniques of computer science. Inside hedge funds as well as banks new computer programs are being created in order to beat algoritms of their competitors. Therefore, it is no surprise that neural networks are being used for assessing options as well as other operations in finance.

The main objective of this work was to suggest a new hybrid prediction model on the base of RBF neural network and ECM (Error-Correction Model or Error-Correction mechanism) for forecasting high-frequentional data; theoreticaly justify the corretness of this model and empirically test this model on real financial data. In our work we suggest, implement, validate Error-Correction model of feedforward neural network, which combines standard neural network and the ECM principle (known from the theory of cointegration) into one complex model. The reason for the selected combination of two different methodologies is the assumption of better forecasting performance as well as other better propeperties such as flash estimates forecasts, user comfort, simplicity of the model. The realization of ECM NN is performed by transforming ECM NN to RBF-SEMA model; the error-correction mechanism is implemented via long-term equlibration of short-term trend using the theory of moving averages. The reason why we decided to use RBF neural network for forecasting financial time series is the fact that according to some studies (Gooijer and Hyndman, 2006), neural network have the biggest potential in forecasting financial series. Moreover, Hill et al. (1994) showed that neural networks work best in connection with high-frequentional data.

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Except for the main objective, in our thesis we also combine other methods of machine learning in order to find out whether it brings some improvement of forecasting qualities. The technique we implement into the NN is the evolutionary approach (EC). In our model, genetic algorithm is used as an optimization technique for adapting weights of NN. Moreover, as Kohonen (1995) showed that non-hierarchical clustering algorithms used in connection with NN can bring better prediction qualities of NN, we also use clustering.

This work applies presented models into financial area - forex, i.e. financial time series of exchange rates. Forex is one of the biggest and most liquid financial markets in the world. The suggested model je verified on high-frequentional data and the prediction abilility is examined via one-day-ahead forecasts. Validation of the suggested and implemented model is exexuted via comparative out-of-sample analysis together with statistical models as well as standard model of NN. Out-of-sample approach was selected due to futute predictability of the model as overfitting is very propable when using in-sample approach (Dacca and Satchell, 1999).

2. Financial Forecasting - Analysis

The most used approach in forecasting financial time series is statistical approach (Gooijer and Hyndman, 2006). In financial area prognostic problems are calculated via econometrics models such as linear regression, Kalman filter, cointegration models. The most popular models in this area are ARIMA models (Box and Jenkins, 1976) as (O'Donovan, 1983) showed that these models provide significantly better forecasts than other models. But later, Engle showed that using ARIMA in financial modelling is not correct and he suggested ARCH models (Engle, 1982). Today, models based on ARCH are the most popular models in finance. Banks use these models to determine risk of their assets using the strategy Value at Risk.

Even though statistical approach is the most dominant techniques of financial forecasting, it does not always provide satisfied results. One of the biggest problem is unsufficient prediction quality of these models when forecasting high-frequentional, very dynamic and high-volatile financial time series. Due to massive explosion of IT, other models are being incorporated into financial forecasting. We call them machine learning models. In recent years artificial neural network have become very popular tool for financial analysts all over the world. The reason why NN are used for this purpose is that a neural network is an universal functional black-box approximator of non-linear type (Hornik, 1989, 1993; Maciel and Ballini, 2008), which is very helpful in modelling non-linear processes which have apriori unknown functional relations (Darbellay a Slama, 2000), moreover, they are able to model even chaotic time series (Zhang and Patuwo, 1998).

Models of NN are applied to many different problems. Demand for electricity modelled using NN is realized in study of Darbellay and Slama (Darbellay and Slama, 2000). Wu (Wu and Hao, 2009) with radial basis NN modelled expenditures on education. Sunaryo (Sunaryo et al., 2011) applies models of recurrent neural networks to forecast prices of electricity. Yao and Tan (2000) apply neural network on exchange rates and they empirically illustrate their ability to handle also financial time series. Qi (2001) used methods of neural networks to predict recession in USA. Zhou (Zhou, 2011) applied NN to model GDP of China. Dunis (2007) predicted the prices of gold and silver using ARIMA models and neural networks. (Boyacioglu et al., 2009) realized a study where they tested statistical models and NN for bank failures. The reason for attractiveness of NN for forecasting time series in financial area can be find in publication of (Maciel a Ballini, 2008). The study states that the phenomenon of attractiveness of NN in forecasting could be related to three reasons. At first, NN are black-box approximator of non-linear type. Second, NN are universal approximator. According to (Hornik, 1993), NN can approximate any continuous function into any accuracy. Third, NN are able to generalize.

Except for this, many research teams focus on hybrid models. There exist many combination of neural networks and econometrics models - combination of NN and ARIMA (Bábel, 2011), combination of HMM and GARCH (Zhuang and Chan, 2004). Dhar and Chou (2001) suggest hybrid model (NN + GA) to predict earnings of companies. Yu (2007) in his book present hybrid models of NN combined with genetic algorithms, ARIMA models or exponential smoothing. Šterba and Hiľovská (Šterba and Hiľovská, 2010) in their paper suggested hybrid model NN + ARIMA which they apply to time series in hydrology.

3. Main Objective and Hypothesis

Problem specification

Even though statistical approach is the most dominant approach in financial forecasting today, it does not always provide satisfied results. One of the main problems of this approach in financial domain is unsufficient prediction accuracy in forecasting high-frequentional, very dynamic, high-volatile financial time series. The solution could be construction of hybrid models. Hybrid models are created and applied to many different problems due to improving properties of default model. These models that would combine the best properties from several independent models (statistical, soft-computing or others) into one complex system and would provide significantly better prediction qualities are very desirable.

Main objective of dissertation thesis

On the base on analysis of current state in financial time series forecasting suggest a new advanced hybrid model of neural network, combined with other methods of machine learning as well as statistical models, that would provide better prediction qualities than standard models of neural networks in financial time series forecasting; afterwards implement this model and on base of mathematics as well as empirical experiments on real financial data analyse and demonstrate prediction contribution of suggested model.

Besides this, in our work also other types of hybrid models of neural networks were created due to intention to better prediction properties of neural networks. On base of analysis and the main objective of the thesis we defined four hypothesis:

Hypothesis H1: Hybrid model of NN based on combination of supervised and unsupervised learn-

ing will bring a hybrid model that is faster, more effective, more accurate than the standard model of NN in the area of financial forecasting.

Back-propagation which is used in the learning phase of RBF network has many disadvantages. One of them is for example scaling problem, other one is convergence which is very slow. Due to these and many other reasons we decised to use and implement combined learning technique for adapting parameters of suggested RBF neural network as Kohonen (1995) demonstrated that non-hierarchical clustering algorithms used with NN can bring better predictions than RBF itself. Hybrid learning technique will be composed of combination of back-propagation and standard unsupervised learning technique K-means. K-means is used in the phase of non-random initialization of weight vector w, just before back-propagation.

Hypothesis H2: Algorithmic hybridization of RBF neural network combined with evolutionary approach will bring faster, more effective and more accurate version of prognostic model than standard RBF neural network in the domain of financial time series forecasting.

As genetic algorithms have become a popular optimizing tool in recent years, they will be also used in our solution of combined NN. Evolutionary approach is very often used as a tool of optimization in many areas (Brock et al., 1992). EC is also used in the area of technical analysis (Papadamou and Stephanides, 2007). It is obvious that genetic algoritms can be implemented into NN in many ways. In this work we test whether evolutionary approach used instead of back-propagation in the phase of network learning is better learning technique than the standard algorithm as according to theory, GA would not have the same scaling problems than BP.

Hypothesis H3: Suggested and implemented Error-Correction model of RBF neural network with implemented error-correction mechanism will better prediction accuracy of this model versus standard model of neural network.

The main properties of this new constructed hybrid model is except for gaining better prediction accuracy also flash estimates, user comfort, simplicity of the models for a user. The reason why we decided to use RBF neural network for forecasting financial time series is the fact that according to some studies (Gooijer and Hyndman, 2006), neural network have the biggest potential in forecasting financial series. Moreover, Hill et al. (1994) showed that neural networks work best in connection with high-frequentional data.

Hypothesis H4: By performed comparative analysis of classical statistical prediction model for forecasting financial time series, soft-computing models and hybrid models one can point out that hybrid soft-computing models provde significantly better prediction qualities than standard statistical models.

When evaluating prediction accuracy of hybrid models we want, except for their mutual comparison, realize a procedure suggested in (Adya a Collopy, 1998) and compare our hybrid models with standard statistical models, which can be used as a benchmark. The validity of hypothesis is verified on real financial data of selected exchange rates. Prediction ability is investigated on one-day horizon. Validation of our suggested models is realized via out-of-sample approach mainly due to overfitting problem in in-sample-analysis.

3.1 Methodology

We used more methodologies in our work. We first used Box-Jenkins methodology that was used to model the time series with ARIMA statistical models. Methodology of empirical modelling used f.ex. in (Marcek, 2009) was used due to prognostic verification of suggested models. Except for this, for experiment evaluation we used methodology used in Heider et al. (2010). Moreover, due to correct progress we also suggested and created own methodologies:

Methodology of exchange rates selection was used when chosing exchange rates that were used in empirical research. As there is many exchange rates, it was not possible to realize modelling with all of them. Due to that we had to choose the correct and the most representatives exchange rates. Methodology of forecasting with ECM NN which comes from cointegration Engle-Granger model and is customized to neural network. This methodology is rather theoretical and general. Methodology of RBF-SEMA forecasting comes from the previous one and states the exact practical procedure how to realize modelling and forecasting of time series using hybrid model created in our thesis. Methodology of NN forecasting states the exact procedure how to forecasting time series with our implemented RBF neural network.

4. New Advanced Hybrid Models of Neural Networks

We decided to use hybrid modelling due to more reasons. One of them is the possibility of better prediction qualities of these models. Studies of Yang (2000) and Clemen (1989) theoretically proved that a combination of more models can bring better and more accurate forecast. Due to that we also combine neural network of RBF type with other models. We selected feedforward neural network as our default model as according to Cybenko theorem (Cybenko, 1989), neural networks are universal approximator that is able to approximate any continuous function.

4.1 Implementation of Unsupervised Learning into ANN

Suggested Hybrid Model RBF-KM

In this work we combine RBF with standard clustering algorithm K-means. The main thing is to define k centroids, one for every cluster. The method than produces exactly k clusters and the addition of squares of distances in the segment is minimal. Firstly, algorithms randomly choses coordinances of k centres, from which every represent a centre of a segment. Other objects are allocated to segments on the base of similarity.

K-means is used in the phase of non-random initialization of weight vector w realized before learning of the network. In many cases it is not necessary to interpolize output value by radial function, it is just sufficient to use one function to some set of data (cluster), whose center is considerer to be a center of an activation function of neuron. Algorithm have four steps, we used the adaptive version which can be find in the dissertation thesis.

4.2 Implementation of Unsupervised Learning into ANN

Suggested Hybrid Model RBF-GA

We decided to combine evolutionary approach into our model of RBF neural network as an alternative algorithms for adapting weights of the network, i.e. for searching optimal parameters of neural network. The reason for incorporating GA into NN is the fact that GA should not have the same problem with scaling as BP, as GA improves the current solution monotolically. GA are also very good for solution of problems where the objective function is discontinuous, non-differentiable, non-convex or noisy.

To create specific GA for model of our RBF neural network, more parameter is needed: method of coding chromosomes, fitness or evaluating function used for evaluating chromosomes, population size, initialization sample, method applied on operation of crossover and method applied on operation mutation. The implementation of GA into RBF come from (Davis, 1987) and for our RBF neural network is defind as follows:

1. **The way of coding chromosomes** - specific gene in the chromosome of this GA is a float value and represent a specific weight of a synapse in the network. The whole chromosome represent weights of the whole neural network. The length of a chromosome is then d = D*s - s, where s is the number of neuron in the hidden layer and D is the dimension of the input vector.

2. **Evalutation function which return evaluation value for every chromosome** - as the main criteria is the prediction quality of models, we selected Mean Square Error as the evaluation function

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(y_t - \hat{y}_t)^2 = \frac{1}{N}\sum_{t=1}^{N} e_t^2 \qquad (1)$$

3. **The way of initializing a population of chromosomes** - the individual with the lowest value of MSE will be automatically transferred into the next generation. Other individuals of the next generation are selected as follows - by tournament selection individuals are chosen randomly (we used the sample of tournament size 100) from the current population. The individual with the lowest value of MSE is the first parant. The second parent is selected the same way.

4. **Operators applied in the reproduction process** - new individuals are created by the crossover operation from two selected parents according to following algorithm:

   IF $generated\_value < 0,5$

   THEN write the weight of 1^{st} parent of the specific position of the new individual

   ELSE write the weight of 2^{nd} parent of the specific position of the new individual

   The value is generated from the interval $\langle 0, 1)$. Coefficient of mutation was set to 0.01. If the mutation was performed, the specific gene was set to random value.

5. **Setup for the algorithm** - will be discussed later in the paper.

4.3 Suggested New Error-Correction Model of Neural Network

Presentation of ECM Model of NN

When defining our Error-Correction model of RBF we will come from Engle-Granger version of ECM cointegration model (Engle and Granger, 1987). Let y_t be a time series and let x_t be a time series that is unknown and that is in a linear combination with y_t and which has an "influence" on the series y_t. The relation can be formally defined as

$$y_t = \alpha + \beta x_t + z_t \qquad (2)$$

and the error of the first regression (the default NN) is defined as

$$z_t = y_t - (\alpha + \beta x_t) \qquad (3)$$

Then, the regression of addition of y_t (i.e. differences) to lagged changes of time series x_t as well as balanced errors represented by z_t is

$$\Delta y_t = \beta_0 \Delta x_{t-1} - \beta_1 z_{t-1} \qquad (4)$$

Let w_j be a series of exogenous shocks that influence the series y_t. Then, the relation (2) and. (3) can be written as

$$y_t = \alpha + \beta x_t + \beta w_{1t} + \beta w_{2t} + \cdots + \beta w_{it} + z_t \qquad (5)$$

$$z_t = y_t - (\alpha + \beta x_t + \beta w_{1t} + \beta w_{2t} + \cdots + \beta w_{it}) \qquad (6)$$

From (4), (5) and (6) we can deduce the general form of Error-Correction model of neural network

$$\Delta y_t = \alpha + \beta_0 \Delta x_{t-1} - \beta_1 EC_{t-1} + \epsilon_t \qquad (7)$$

where ϵ_t is the time series of errors from Error-Correction model of NN, where β_0 captures short-term effects of series x_t on series y_t and β_1 captures the speed at which system return to equilibrium state after short-term shock and EC is error-correction mechanism of short-term shocks.

Let define x_t, as a time series that is in linear combination with predicted time series y_t that has an influence of the series y_t, and is a functional generator of non-linear type of RBF feedforward neural network:

$$x_t = \sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}} \qquad (8)$$

where v_j are weights coming from the hidden neurons to the output value and where w_i^j represent weighs of RBF network coming from the input layer to the hidden layer, where xx is the vector of inputs coming to the NN and where σ_j^2 is the variance of the selected cluster i defined by the weights w_i and xx.

It is important to note that time series generated by RNF network is a time series that includes short-term shocks from the equilibrium. Moreover, as NN is a predictor of the original series, there exists an assumption that the series x_t is cointegrated with the series y_t. Let define y_t

as a time series we want to forecast, i.e. as a desired output of our model. Let define z_t, i.e. the error of the first regression as the error of the original RBF network. In that case, the relations (5) and (6) can be written as

$$y_t = \alpha + \beta \sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}} + \epsilon_t^{RBF} \quad (9)$$

where

$$\epsilon_t^{RBF} = z_t = y_t - (\alpha + \beta \sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}}) \quad (10)$$

Then, the relation (7), i.e. regression of he equation of increment y_t (i.e. differences) on lagged changes of time series x_t generated by RBF as well as balanced errors represented by ϵ_t^{RBF} can be expressed as

$$\Delta y_t = \beta_0 \Delta(\sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}})_{t-1} - \beta_1 \epsilon_{t-1}^{RBF} \quad (11)$$

Let e_i is the series of exogenous external shocks that influence the series y_t. Then the relation (9) and (10) can be analogically according to (5) and (6) expressed as

$$y_t = \alpha + \beta(\sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}})_t + \\ + \beta e_{1t} + \beta e_{2t} + \dots + \beta e_{it} + \epsilon_t^{RBF} \quad (12)$$

$$\epsilon_t^{RBF} = y_t - (\alpha + \beta(\sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}})_t + \\ + \beta e_{1t} + \beta e_{2t} + \dots + \beta e_{it}) \quad (13)$$

We can define the basic Error-Correction model od RBF from equations (11) to (13):

$$\Delta y_t = \alpha + \beta_0 \Delta(\sum_{j=1}^{s} v_j * \exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}})_{t-1} - \\ - \beta_1 EC_{t-1} + \epsilon_t^{ECM} \quad (14)$$

where ϵ_t is the time series of errors from Error-Correction model of NN, where β_0 captures short-term effects of series x_t on series y_t and β_1 captures the speed at which system return to equilibrium state after short-term shock and EC is error-correction mechanism of short-term shocks that represent long-term trend. This EC term is in ECM NN defined as

$$EC_t = \frac{\sum_{i=1}^{n} e_i^{RBF}}{n} \quad (15)$$

Where e_i is the time series of residuals of the original model of NN (without EC term).

On base of the relations defined above, we can use the following formula to compute the future value

$$y_t = y_{t-1} + \Delta y_t \quad (16)$$

Where Δy_t has implemented EC mechanism, i.e. it is a response on short-term shock of x_t and where UNS (x_t) is the short-term predictor (current) and where EC (moving average) repairs these shocks by long-term trend (moving average).

The main of this suggested ECM NN is that cointegrated time series share stochastic component and long-term equalled state. Deviances from this long-term state are the result of short-term shock that are repaired by the error-correction mechanism. This model is very helpful for data with long-term memory. Financial time series are this kind of data as they exhibit characteristics of high volatility.

Transformation of ECM of NN to Computational RBF-SEMA Model

RBF-SEMA computational model (Radial Basis Function - Simple Error Moving Average) is a hybrid model, that is a transformation of ECM NN.

Let G be a restriction of function F: $x_t \in \mathbb{R}^k \rightarrow y_t \in \mathbb{R}^1$ (F is a functional representation assigning one value y_t to n-dimensional input in the given time period) that G(x_t, w_t, v_t, s) : $x_t \in \mathbb{R}_{train}^k \rightarrow y_t \in \mathbb{R}_{train}^1$, where $\mathbb{R}_{train}$ is a complement $\mathbb{R}_{val}$ into $\mathbb{R}$. Then, the computational model RBF-SEMA (X,W,V,S,q) can be written as

$$G(x_t, w_t, v_t, s) = \psi_2 \sum_{j=1}^{s} (v_j o_j) + \epsilon_t^{RBF} \quad (17)$$

where

$$\epsilon_t^{RBF} = e_t + \epsilon_t^{ECM}, \quad \epsilon_t^{ECM} = u_t \approx iid(0,1) \quad (18)$$

and

$$e_t = \sum_{i=1}^{q} \theta \epsilon_{t-i}^{RBF} \quad (19)$$

and

$$\theta_1 = \theta_2 = \theta_3 = \dots = \theta_q = 1/q \sum_{i=1}^{q} \theta_i = 1 \quad (20)$$

and where

$$o_j = \psi_1[\phi(x, w^j)], j = 1, 2, ..., s \quad (21)$$

where ϕ is a radial basis function

$$\phi(x, w) = \phi(||x - w||) \quad (22)$$

and where ψ_1 is a Gauiis function for activation of j hideen neurons defined as

$$\psi_1(u^j) = e^{\frac{-u^j}{2\sigma_j^2}} = e^{\frac{-||x-w^j||^2}{2\sigma_j^2}}, j = 1, 2, \dots s \quad (23)$$

where σ_j^2 is the variance of j^{th} hidden neuron and u is the potential of this neuron. Moreover

$$\psi_2(u) = u = \sum_{j=1}^{s} v_j o_j \quad (24)$$

Also, for the whole model the necessary condition must be met

$$E(w_t) = \sum_{x_t, y_t \in R_{train}^k} ([G(x_t, w_t, v_t, s) + \epsilon_t^{RBF}] - y_t)^2 \rightarrow \min \quad (25)$$

i.e. model must be adapted to the unknown function F, model must meet the basic condition that the difference between the estimated output and the default output must be minimal.

Also $X = (x_1, x_2, x_3, x_4, \ldots x_n)$ is the input layer of the network and is also a vector of lagged autoregressive part and n is the order of autoregression.

Mathematical Justification of Application of ECM of NN

We prove that if the suggested ECM NN exists, it has always lower value of error than the standard NN. The existence of hybrid model is proved by existence of parameter N for EC mechanism. Let define

$$MSE_{UNS} = \frac{1}{N} \sum_{1}^{n} e_t^2, \tag{26}$$

$$e_t = y - \hat{y}_{UNS} \tag{27}$$

$$MSE_{ECM\_UNS} = \frac{1}{N} \sum_{n-N}^{n} e_t'^2 \tag{28}$$

$$e_t' = y - \hat{y}_{ECM\_UNS} \tag{29}$$

$$\hat{y}_{ECM\_UNS} = \hat{y}_{UNS} + \frac{e_{t-1}^2 + e_{t-2}^2 + \cdots + e_{t-N}^2}{N} \tag{30}$$

where $\hat{y}_{UNS}$ is the estimation of the standard NN, $\hat{y}_{ECM\_UNS}$ is the estimation of the ECM NN, y is the original real value in time t, e_t is the error of the standard NN and e_t' is the error of the ECM NN. Let define N as the parameter of error-correction mechanism, i.e. the value of moving average parameter. Let assume that n (number of observations) is large enough to determine the right value of parameter N. Then, our goal is to find out if exist N, for which the following inequation is right

$$?\exists N \in N, MSE_{ECM\_UNS} < MSE_{UNS} \tag{31}$$

$$\frac{e_{N+1}'^2 + e_{N+2}'^2 + \cdots + e_n'^2}{n-N} < \frac{e_1^2 + e_2^2 + \cdots + e_n^2}{n} \tag{32}$$

Let define m = N + 1. After several algebraic operation we get the following inequation:

$$\frac{1}{N} < \frac{2(y_m - \hat{y}_{m\_UNS})(y_{m-1} - \hat{y}_{m-1\_UNS} + \ldots}{(y_{m-1} - \hat{y}_{m-1\_UNS} + \cdots + y_1 - \hat{y}_{1\_UNS})^2 + \ldots}$$

$$\frac{\cdots + y_1 - \hat{y}_{1\_UNS}) + \cdots + 2(y_n - \hat{y}_{n\_UNS})}{\cdots + (y_{n-1} - \hat{y}_{n-1\_UNS} + \ldots}$$

$$\frac{(y_{n-1} - \hat{y}_{n-1\_UNS} + \cdots + y_{n-N} - \hat{y}_{n-N\_UNS})}{\cdots + y_{n-N} - y_{n-N\_UNS})^2} \tag{33}$$

Unfortunataly, it is not possible to generalize this inequation any more and express N. In mathematics, problems like this are calculated numerically. We verify for which N the inequation is true (we test all N from our set), we compare which of them has the lowest MSE and we then get our N.

We proved mathematically the condition of existence of our hybrid model. Our hybrid model exists if exists at least one N for which the inequation (33) is right. If there is at least one N for which this inequation is true, we can state that hybrid ECM NN model exists and is always better and has lower MSE than the standard NN.

5. Experiments

For practical verification of our hypothesis we chose financial time series. This was done due to the fact that they exhibit properties such as high dynamics, they are noisy, chaotic, unstable, volatile, high frequentional etc. We chose forex data - exchange rates. The first exchange rate - EUR/USD we chose due to the fact that it is the most traded exchange rate in the world. Moreover, this pair is very volatile, dynamic, unstable. The second exchange rate we selected is AUD/USD. This pair was selected on base of cross-corellation analysis where we found out that this pair is the most correlated pair with other major pairs. Due to that there is an assumption that is we can forecast this pair with high accuracy, we will be able to forecast well also other correlated pairs.

5.1 Methodology of Empirical Modeling

For AUD/USD we used the following interval (10/31/2008 - 10/31/2012). The series was divided into two parts - the first part (10/31/2008 - 4/30/2012) was used for model training, the second part (5/1/2012 - 10/31/2012) was used for model validation. For EUR/USD we used the following interval (1/1/2009 - 12/31/2013) for training and 1/1/2014 - 31/12/2014 was used for validating the model.

5.2 Realization of Experiments

We tested the network with the number of processing neurons varying from 3 to 10. Due to the elimination of deformation of results we used the procedure suggested in Heider et al. (2010). Due to comparative analysis of our models we defined following numerical characteristics: average approximation ability (average MSE in training set), average prediction ability (average MSE in validation set), consistency of predictions (standard deviation), maximal prediction power (the lowest MSE in the given tested configuration).

5.3 Experiment Evaluation

Hypothesis H1 Evaluation

Hypothesis H1: Hybrid model of NN based on combination of supervised and unsupervised learning will bring a hybrid model that is faster, more effective, more accurate than the standard model of NN in the area of financial forecasting.

RBF-KM model on the validation set made worse values of AMSE in none of 16 testing configurations. In 50% of tests it improved AMSE 2 to 50 times. Standard deviation was lowered in 100 percent of tests and in 44 percent it was lowered 10 times and in more than 30 percent of test it was lowered 100 times. We can state that the use of K-means in RBF model causes improvement in speed as well as accuracy of the network. The biggest strength of this combination is consistency of predictions.

Conclusion of hypothesis H1: Hybrid model of NN based on combination of supervised and unsupervised learning WILL BRING a hybrid model that is faster, more effective, more accurate than the standard model of NN in the area of financial forecasting.

Table 1: Comparison of Prediction Accuracy (Ex-post, AUD/USD)

Inputs	Neurons	Model	Data: AUD/USD (1/5/2012 - 31/10/2012)		
			LMSE	AMSE	sd
autoregressive(1)	3	RBF	0,0000352932	0,0000371938	0,0000016322
		RBF-KM	0,0000352387	0,0000359097	0,0000006952
		RBF-GA	0,0000350804	0,0000394612	0,0000077576
	4	RBFl	0,0000354141	0,0000361420	0,0000010502
		RBF-KM	0,0000351368	0,0000359361	0,0000005746
		RBF-GA	0,0000352928	0,0000388879	0,0000056151
	5	RBF	0,0000353457	0,0000383285	0,0000029750
		RBF-KM	0,0000353685	0,0000365858	0,0000011527
		RBF-GA	0,0000352275	0,0000450899	0,0000172736
	6	RBF	0,0000356998	0,0000645906	0,0000487982
		RBF-KM	0,0000356588	0,0000376993	0,0000023022
		RBF-GA	0,0000352464	0,0000699166	0,0000210699
	7	RBF	0,0000350679	0,0000380268	0,0000024547
		RBF-KM	0,0000349884	0,0000368605	0,0000015725
		RBF-GA	0,0000353884	0,0000617597	0,0000409291
	8	RBF	0,0000350225	0,0000403603	0,0000050314
		RBF-KM	0,0000357058	0,0000385444	0,0000018808
		RBF-GA	0,0000385785	0,0001255561	0,0000990258
	9	RBF	0,0000350950	0,0000417112	0,0000071166
		RBF-KM	0,0000360165	0,0000410658	0,0000050695
		RBF-GA	0,0000394757	0,0000918037	0,0000591737
	10	RBF	0,0000351339	0,0000439533	0,0000038039
		RBF-KM	0,0000355041	0,0000396413	0,0000045812
		RBF-GA	0,0000437501	0,0001310440	0,0000579103

AMSE average of MSE
LMSE lowest MSE from the 12 replications
sd standard deviation

Table 2: Comparison of Prediction Accuracy (Ex-post, AUD/USD)

Inputs	Neurons	Model	Data: AUD/USD (1/5/2012 - 31/10/2012)		
			LMSE	AMSE	sd
autoregressive(1)	3	RBF	0,0000253270	0,0000696935	0,0001172497
		RBF-KM	0,0000254342	0,0000805217	0,0000471395
		RBF-GA	0,0000243691	0,0000273484	0,0000038203
	4	RBFl	0,0000250646	0,0000289508	0,0000057071
		RBF-KM	0,0000253400	0,0000731818	0,0000531773
		RBF-GA	0,0000246709	0,0000284446	0,0000034659
	5	RBF	0,0000254642	0,0002886271	0,0005325626
		RBF-KM	0,0000251173	0,0000355195	0,0000192745
		RBF-GA	0,0000248531	0,0000293482	0,0000062376
	6	RBF	0,0000251831	0,0005365776	0,0006403063
		RBF-KM	0,0000250912	0,0000293871	0,0000045754
		RBF-GA	0,0000244162	0,0000298348	0,0000028225
	7	RBF	0,0000250294	0,0001568964	0,0001665586
		RBF-KM	0,0000250124	0,0000308270	0,0000058896
		RBF-GA	0,0000253390	0,0000336861	0,0000093192
	8	RBF	0,0000250960	0,0003459688	0,0003921289
		RBF-KM	0,0000249680	0,0000283194	0,0000024171
		RBF-GA	0,0000248105	0,0000517004	0,0000183724
	9	RBF	0,0000250697	0,0004472941	0,0005032039
		RBF-KM	0,0000248607	0,0000257253	0,0000007344
		RBF-GA	0,0000276469	0,0000535409	0,0000269321
	10	RBF	0,0000287856	0,0011711587	0,0004968528
		RBF-KM	0,0000249000	0,0000263059	0,0000023926
		RBF-GA	0,0002965460	0,0000770754	0,0000364193

AMSE average of MSE
LMSE lowest MSE from the 12 replications

Table 3: Prediction Accuracy of NN and ECM NN (Ex-post, AUD/USD)

Neurons	Model	Data: AUD/USD (1/5/2012 - 31/10/2012)					
		NN			ECM (RBF-SEMA)		
		LMSE	AMSE	sd	LMSE	AMSE	sd
3	RBF	0,0000352932	0,0000371938	0,0000016322	0,0000261714	0,0000266874	0,0000005880
	RBF-KM	0,0000352387	0,0000359097	0,0000006952	0,0000261981	0,0000272105	0,0000008639
	RBF-GA	0,0000350804	0,0000394612	0,0000077576	0,0000271588	0,0000325561	0,0000051517
4	RBF	0,0000354141	0,0000361420	0,0000010502	0,0000261802	0,0000265099	0,0000006305
	RBF-KM	0,0000351368	0,0000359361	0,0000005746	0,0000261943	0,0000272980	0,0000010261
	RBF-GA	0,0000352928	0,0000388879	0,0000056151	0,0000265978	0,0000298731	0,0000032434
5	RBF	0,0000353457	0,0000383285	0,0000029750	0,0000261770	0,0000264689	0,0000003574
	RBF-KM	0,0000353685	0,0000365858	0,0000011527	0,0000261388	0,0000277552	0,0000014226
	RBF-GA	0,0000352275	0,0000450899	0,0000172736	0,0000263147	0,0000324241	0,0000041909
6	RBF	0,0000356998	0,0000645906	0,0000487982	0,0000261858	0,0000297946	0,0000041403
	RBF-KM	0,0000356588	0,0000376993	0,0000023022	0,0000261513	0,0000269852	0,0000009301
	RBF-GA	0,0000352464	0,0000699166	0,0000210699	0,0000262808	0,0000353730	0,0000031900
7	RBF	0,0000350679	0,0000380268	0,0000024547	0,0000261802	0,0000280647	0,0000027893
	RBF-KM	0,0000349884	0,0000368605	0,0000015725	0,0000261594	0,0000300232	0,0000053526
	RBF-GA	0,0000353884	0,0000617597	0,0000409291	0,0000262444	0,0000360950	0,0000078080
8	RBF	0,0000350225	0,0000403603	0,0000050314	0,0000261442	0,0000320638	0,0000064355
	RBF-KM	0,0000357058	0,0000385444	0,0000018808	0,0000261346	0,0000272803	0,0000009354
	RBF-GA	0,0000385785	0,0001255561	0,0000990258	0,0000263457	0,0000339704	0,0000042332
9	RBF	0,0000350950	0,0000417112	0,0000071166	0,0000261851	0,0000320811	0,0000054598
	RBF-KM	0,0000360165	0,0000410658	0,0000050695	0,0000261920	0,0000284762	0,0000027145
	RBF-GA	0,0000394757	0,0000918037	0,0000591737	0,0000322298	0,0000559772	0,0000255650
10	RBF	0,0000351339	0,0000439533	0,0000038039	0,0000261680	0,0000297882	0,0000035248
	RBF-KM	0,0000355041	0,0000396413	0,0000045812	0,0000261359	0,0000285398	0,0000031133
	RBF-GA	0,0000437501	0,0001310440	0,0000579103	0,0000350972	0,0000403218	0,0000077488

Table 4: Prediction Accuracy of NN and ECM NN (Ex-post, EUR/USD)

Neurons	Model	Data: EUR/USD (1/1/2014 - 31/12/2014)					
		NN			ECM		
		LMSE	AMSE	sd	LMSE	AMSE	sd
3	RBF	0,0000253270	0,0000696935	0,0001172497	0,0000248508	0,0000263090	0,0000014020
	RBF-KM	0,0000254342	0,0000805217	0,0000471395	0,0000268712	0,0000370237	0,0000085489
	RBF-GA	0,0000243691	0,0000273484	0,0000038203	0,0000245941	0,0000282454	0,0000010255
4	RBF	0,0000250646	0,0000289508	0,0000057071	0,0000253748	0,0000276226	0,0000018085
	RBF-KM	0,0000253400	0,0000731818	0,0000531773	0,0000244467	0,0000370706	0,0000143551
	RBF-GA	0,0000246709	0,0000284446	0,0000034659	0,0000236783	0,0000267542	0,0000012329
5	RBF	0,0000254642	0,0002886271	0,0005325626	0,0000245879	0,0000269191	0,0000017104
	RBF-KM	0,0001123029	0,0000355195	0,0000192745	0,0000245353	0,0000282904	0,0000027106
	RBF-GA	0,0000247523	0,0000293482	0,0000062376	0,0000245863	0,0000279513	0,0000014217
6	RBF	0,0000251831	0,0005365776	0,0006403063	0,0000248786	0,0000259294	0,0000007543
	RBF-KM	0,0000250912	0,0000293871	0,0000045754	0,0000242192	0,0000267968	0,0000032970
	RBF-GA	0,0000244162	0,0000298348	0,0000028225	0,0000245187	0,0000266889	0,0000017984
7	RBF	0,0000250294	0,0001568964	0,0001665586	0,0000247944	0,0000266337	0,0000014758
	RBF-KM	0,0000250124	0,0000308270	0,0000058896	0,0000245911	0,0000289844	0,0000049943
	RBF-GA	0,0000253390	0,0000336861	0,0000093192	0,0000240032	0,0000279224	0,0000027567
8	RBF	0,0000250960	0,0003459688	0,0003921289	0,0000243591	0,0000266844	0,0000013359
	RBF-KM	0,0000249680	0,0000283194	0,0000024171	0,0000238849	0,0000272206	0,0000023014
	RBF-GA	0,0000248105	0,0000517004	0,0000183724	0,0000237238	0,0000267998	0,0000016624
9	RBF	0,0000250697	0,0004472941	0,0005032039	0,0000250388	0,0000259964	0,0000008749
	RBF-KM	0,0000248607	0,0000257253	0,0000007344	0,0000244119	0,0000267900	0,0000015011
	RBF-GA	0,0000276469	0,0000535409	0,0000269321	0,0000236858	0,0000277838	0,0000041275
10	RBF	0,0000287856	0,0011711587	0,0004968528	0,0000246893	0,0000263602	0,0000012951
	RBF-KM	0,0000249000	0,0000263059	0,0000023926	0,0000244957	0,0000259175	0,0000014823
	RBF-GA	0,0002965460	0,0000770754	0,0000364193	0,0000239551	0,0000303116	0,0000068579

Table 5: Improvement of RBF-SEMA ECM NN Versus the Standard NN (Ex-post)

Series	Neurons	Percentual improvement of MSEE versus standard NN [v %]		
		RBF-SEMA vs RBF	RBF_{KM}-SEMA vs RBF_{KM}	RBF_{GA}-SEMA vs RBF_{GA}
AUD/USD	3	-28,25%	-24,23%	-17,50%
	4	-26,65%	-24,04%	-23,18%
	5	-30,94%	-24,14%	-28,09%
	6	-53,87%	-28,42%	-49,40%
	7	-26,20%	-18,55%	-41,56%
	8	-20,56%	-29,22%	-72,944%
	9	-23,09%	-30,66%	-39,03%
	10	-32,23%	-28,00%	-69,23%
EUR/USD	3	-62,25%	-54,02%	+3,27%
	4	-4,58%	-49,34%	-5,94%
	5	-90,67%	-20,35%	-4,75%
	6	-95,16%	-8,81%	-10,54%
	7	-83,04%	-5,97%	-17,11%
	8	-92,28%	-3,88%	-48,16%
	9	-94,18%	+4,13%	-48,11%
	10	-97,75%	-1,47%	-60,67%

Table 6: Numerical Comparison of Prediction Qualities (Ex-post)

series	model	optimization	$LMSE_{OPT}$
AUD/USD	RBF_{KM}-SEMA	K-means + BP	0,0000261346
	RBF-SEMA	back-propagation	0,0000261442
	$RBF_{GA} - SEMA$	Genetic algorithm	0,0000262444
	RBF-KM	K-means + BP	0,0000349884
	RBF	back-propagation	0,0000350225
	RBF-GA	Genetic algorithm	0,0000350804
	SVR	linear kernel	0,0000352817
	AR(0)+PGARCH (1,1,1)	MNÅă + Marquardt	0,0000358500
	RWP	MNÅă	0,0000359161
	GARCH(1,1)	MNÅă + Marquardt	0,0000359520
EUR/USD	$RBF_{GA} - SEMA$	Genetic algorithm	0,0000236783
	$RBF_{KM} - SEMA$	K-means + BP	0,0000238849
	RBF-SEMA	back-propagation	0,0000243591
	RBF-GA	Genetic algorithm	0,0000243691
	AR(3)+ARCH(7)	MNÅă + Marquardt	0,0000246313
	AR(3)+GARCH (1,1)	MNÅă + Marquardt	0,0000246700
	RWP	MNÅă	0,0000247406
	RBF-KM	K-means + bp	0,0000248607
	RBF	back-propagation	0,0000250294
	SVR	linear kernel	0,0000252125

Hypothesis H2 Evaluation

Hypothesis H2: Algorithmic hybridization of RBF neural network combined with evolutionary approach will bring faster, more effective and more accurate version of prognostic model than standard RBF neural network in the domain of financial time series forecasting.

In general we caon constate that model RBF-GA made better AMSE prediction in 50 percent of tests (in 40 percent the values of AMSE were better 10 times). In 19 percent of tests, the model made worse results. In 75 percent of tests the standard deviation made comparable results. Values of LMSE were better in 56 percent of tests. Looking at table 1 and 2 we can state that GA is much more effective in EUR/USD series. In AUD/USD series it made the comparable results with RBF. The question is if it is due to bad configuration or it is the GA itself. We come back to this question in discussion. When comparing AMSE of RBF-KM and RBF-GA, in 80 percent of test the results were comparable. The convergence of GA is much more faster than the convergence of BP.

Conclusion of Hypothesis H2: On base of performed tests we are not able to say whether algorithmic hybridization of RBF neural network combined with evolutionary approach will bring faster, more effective and more accurate version of prognostic model than standard RBF neural network in the domain of financial time series forecasting.

Hypothesis H3 Evaluation

Hypothesis H3: Suggested and implemented Error-Correction model of RBF neural network with implemented error-correction mechanism

Table 7: Analysis of Prediction Contribution Made by Increased Number of Generations (RBF-GA)

		10 generations		20 generations		
AUD/USD	Neurons	MSE_A	MSE_E	MSE_A	MSE_E	% improvement (ex-post)
	3	0,0000838781	0,0000394612	0,0000769570	0,0000362380	-8,17
	4	0,0000863849	0,0000388879	0,0000884140	0,0000407563	+4,80
	5	0,0000914001	0,0000450899	0,0001164652	0,0000406821	-9,78
	6	0,0001186774	0,0000699166	0,0000826175	0,0000435280	-37,74
	7	0,0001380118	0,0000617597	0,0000850205	0,0000418516	-32,23
	8	0,0002033276	0,0001255561	0,0000865789	0,0000449178	-64,22
	9	0,0002061880	0,0000918037	0,0001322653	0,0000479574	-47,76
	10	0,0002389402	0,0001310440	0,0003008191	0,0000495256	-62,21
EUR/USD	3	0,0000771487	0,0000273484	0,0000764695	0,0000256231	-6,31
	4	0,0000793920	0,0000284446	0,0000763599	0,0000256979	-9,66
	5	0,0000816416	0,0000293482	0,0000766969	0,0000258482	-11,93
	6	0,0000838296	0,0000298348	0,0000770779	0,0000269325	-9,73
	7	0,0000978521	0,0000336861	0,0000809997	0,0000287289	-14,72
	8	0,0001073335	0,0000517004	0,0000854296	0,0000355817	-31,18
	9	0,0001034963	0,0000535409	0,0000864304	0,0000364606	-31,90
	10	0,0001589123	0,0000770754	0,0000888059	0,0000396124	-48,61

Table 8: Analysis of Prediction Contribution Made by Increased Number of Generations (RBF-GA)

	Population: 1000 (20g)		Population: 5000 (20g)		%improvement
Replication	MSE_A	MSE_E	MSE_A	MSE_E	(ex-post)
1	0,0000763022	0,0000359874	0,0000752470	0,0000355973	-1,08
2	0,0000783971	0,0000391581	0,0000750228	0,0000356453	-8,97
3	0,0000763982	0,0000382620	0,0000750142	0,0000360170	-5,87
4	0,0000772747	0,0000360488	0,0000769981	0,0000387959	+7,62
5	0,0000949480	0,0000491297	0,0000752150	0,0000356117	-27,51
6	0,0004127712	0,0000361168	0,0000749422	0,0000357245	-1,09
7	0,0001113649	0,0000466537	0,0000775278	0,0000401234	-14,00
8	0,0000765381	0,0000369610	0,0000755987	0,0000360983	-2,33
9	0,0000844778	0,0000356248	0,0000749626	0,0000354474	-0,50
10	0,0000761800	0,0000528784	0,0000752503	0,0000351711	-33,49
11	0,0021139271	0,0015397686	0,0000795439	0,0000447725	-97,09
12	0,0000750915	0,0000369610	0,0000757854	0,0000364580	-1,36
PMSE	0,0002794726	0,0001652959	0,0000759257	0,0000371219	-77,54

will better prediction accuracy versus standard model of neural network.

We can constate that in testing configurations RBF-SEMA model bettered AMSE error (compared to RBF) about 47 percent. RBF-KM-SEMA made predictions that were 45 percent better and RBF-GA-SEMA made prediction that had 31 percent lower MSE. Standard deviation improved more than 10 times in 75 percent of tests in RBF-SEMA. In RBF-KM-SEMA sd stayed the same in 88 percent of predictions. In RBF-GA-SEMA sd stayed the same in 50 percent of experiments, in 50 percent sd was 10 times lower.

Conclusion of Hypothesis H3: On base of performed experiments on can say that suggested and implemented Error-Correction model of RBF neural network with implemented error-correction mechanism will better prediction accuracy versus standard model of neural network.

Hypothesis H4 Evaluation

Hypothesis H4: By performed comparative analysis of classical statistical prediction model for forecasting financial time series, soft-computing models and hybrid models one can point out that hybrid soft-computing models provide significantly better prediction qualities than standard statistical models.

In order to compare the effectiveness of suggested hybrid models we performed comparative analysis with statistical models, standard NN models. Except for standard NN, we also tested RBF-KM and RBF-GA model as well as all types of hybrid ECM models.

In both time series, the best results were made with hybrid ECM models. In AUD/USD statistical models were on the last positions. In EUR/USD statistical models made a little bit better result but in general we can say that statistical models had provided significantly worse results than our suggested and implemented hybrid models.

Conclusion of Hypothesis H4: By performed comparative analysis of classical statistical prediction model for forecasting financial time series, soft-computing models and hybrid models one can point out that hybrid soft-computing models REALLY PROVIDE significantly better prediction qualities than standard statistical models.

6. Discussion

RBF-KM model that combined standard learning algorithm with unsupervised clustering algorithm K-means provided better results than the standard RBF neural network in all evaluation criteria. In experiments we found out that RBF-KM provided better or at least comparable prediction quality in 89 per cent of all tests and the prediction of RBF-KM were 9.31 per cent more accurate than standard RBF. On validation set the average improvement was 11.7 per cent and in 50 per cent of tests RBF-KM improved AMSE from 2 to 50 times. Except for higher prediction ability another advantage was consistency of predictions. The standard deviation of predictions decreased (or stayed at least the same) in 89 per cent of tests. Another advantage of this model is convergence of the model.

RBF-GA model with implemented genetic algorithm made more accurate (or at least comparable) predictions in 73 per cent of all tests and the average improvement versus RBF model was 3.9 per cent. Lower values of percentual improvement is, according to us, caused by unsufficient settings of GA in series AUD/USD. However, it is important to note that computational demands were set equally to back-propagation algorithm, unfortunately at AUD/USD this was not sufficient setting. Due to that settings of "better" parameters would be appropriate. If we did not take into account the results from AUD/ISD, the RBF-GA model would improve predictions versus RBF in 97 per cent of tests and the average prediction accuracy would be 35.93 per cent higher.

As in evaluation of hypothesis H2 we were not able to say if the hybridization of GA with RBF would bring more effective and more accurate model of neural network, we come back to this now. In past experiments we set GA to 10 generations and population size was equalled to 1000 (this was done due to computational equivalence with BP). In our additional testing, we decided to raise parameters of GA. We chose two ways - first way was to increase the number of generations from ten to twenty. The second way was to increase the population size (we increased the population size from 1000 to 5000).

From the above tests we can constate that when we increased the parameters of GA, this algorithm increased hugely its prediction qualities. When we increased the number of generation from 10 to 20, the accuracy of predictions was 26.33 per cent higher. When we increased the population size from 1000 to 5000, the prediction accuracy was 77.54 per cent higher. We can hence assume that at 40 or 50 generations or larger population size (10000, 20000) the accuracy of the model would be even more higher. Therefore we can state that H2 hypothesis is true and is valid.

If we compare RBF-GA and RBF-KM (without AUD / USD series) we can say that predictions of RBF-GA were better 3.35 per cent than RBF-KM. If we compare standard deviations, the sd in RBF-KM was 1.85 times lower than sd at RBF-GA. So, in general we can say that RBF-KM made predictions of about same quality, however sd was much more better than at RBF-GA. RBF-GA model made worse predictions with higher number of hidden neurons (due to low number of generations). However, according to us GA has higher potential in forecasting better predictions.

Conclusion about new suggest ECM model

The suggested and implemented ECM model made better prediction in ex-post set in 96 per cent of all experiments. AMSE of ECM was 41 per cent lower compared to standard model. Standard deviation improved (or stayed at least the same) in 100 per cent of tests; in 46 per cent of tests the standard deviation was 10 times lower. LMSE of ECM was 14.21 per cent lower than LMSE of the standard neural network. In general, we can say that ECM provided excellent results and its statistics outperformed all standard models.

In our experiments we found some interesting properties of ECM model. By analyzing experiments on ex-post we found out there is a big consistency of predictions independent of the number of hidden neurons in network infrastructure. We assume that is due to the fact that ECM NN is a robust and scaling model.

Another interesting fact is the non-linearity of improving MSE of ECM NN versus the standard network. At AUD/USD and RBF model (8 hidden neurons), the improvement of ECM NN was more than 72 per cent, at EUR/USD (10 neurons) the improvement was 97.75 per cent. On the contrary, at EUR/USD and GA (3 hidden neurons) the improvement was only 3 per cent versus the standard network. We can say that the bigger the error of the standard RBF, the bigger the repair of ECM NN was. The error is decreasing nonlinearly and seems to be rather geometric or exponential.

Potential Weaknesses of Suggested Model

Of course, we are aware that our implemented solution does not have to be 100 correct. A potential disadvantage of our implementation is that we calculated the AMSE only by averaging 10 replications. It would be more appropriate if we could average 100 replications, however as the length of one replication was from 2 to 5 minutes, the length of all experiments would be much more higher (cca from 400 to 1000 hours).

Future Research

In evaluating hypothesis H2 we found out that in some cases GA produced inconsistent predictions. This problem could be solved (except for increasing the number of generations) also in another way - by implementing hybrid learning algorithms composed of K-means and GA. The reason for doing this is that GA has definitely a bigger potential than BP.

Except for this, it could be possible to suggest cyclical ECM, which would repair the errors of the previous model more than once. An ECM NN from ECN NN would be created. Formally it could be defined as

$$\Delta y_t = \alpha + \beta_0 \Delta (\sum_{j=1}^{s} v_j * exp^{\frac{-\sqrt{\sum_{i=1}^{k}(xx_i^t - w_i^j)^2}^2}{2\sigma_j^2}})_{t-1} - \\ - \beta_1 EC_{t-1}^k + \epsilon_t^{ECM(k)}, \quad k = 1, \ldots, m \quad (34)$$

where m is the order of cyclicity of ECM NN and where

$$EC_t^k = EC_t^{k-1} + \epsilon_t^{ECM(k-1)} \quad (35)$$

Another potential upgrade could be creating advanced versions of RBF-SEMA. Sometimes, it is good to add to the ECM model a dynamic version of error-correction mechanism. Sometimes, new observations have bigger impact on the future value than older observations. Due to that, in our thesis we also suggested and created advanced versions of Error-Correction model of neural network - WECM (Weighted Error-Correction model of NN) and EECM (Exponential Error-Correction model of NN).

7. Theoretical and Practical Contribution of the Thesis

Theoretical contributions:

- Presentation of new complex hybrid Error-Correction model of neural network.
- Theoretical proof of applicability of our suggested ECM NN model by using mathematics.
- Creating own theoretical methodology for modelling and forecasting time series using suggested theoretical ECM NN model.
- Construction of new hybrid statistical+softcomputing models ARCH-RBF for volatility forecasting which is very frequent in financial time series.
- New methodology for forecasting time series with RBF neural network.
- New methodology for forecasting time series with hybrid RBF-SEMA neural network.
- Construction of new hybrid models with non-standard learning algorithm: construction of RBF-KM model combining supervised and unsupervised learning as well as construction of RBF-GA model combining RBF with genetic algorithm.
- Analysis of current state in the area of time series forecasting.
- Presentation of ECM and NN approaches of how to create hybrid models.
- Suggestion of potential improvement of our suggested ECM NN (cyclical ECM NN, weighted ECM NN, exponential ECM NN, ...).

Practical contributions:

- Own programmed application for forecasting time series with RBF neural networks and hybrid neural networks implemented in JAVA programming language.
- Programming implementation of other than standard learning algorithms of neural network.
- Programming implementation of computational version of ECM NN (RBF-SEMA, RBF-KM-SEMA, RBF-GA-SEMA).
- Statistical modelling of selected financial time series which can be used as a guide for statistical modelling in Eviews software.
- Practical contribution of prediction performace realized in real financial data.
- Practical comparison of nonstandard optimization algorithms
- Analysis of performance of genetic algorithm

Creation of theoretical as well as computational model ECM of NN is considered to be the main and biggest contribution of this work. This model is novel in two ways - it combines the theory of econometrics (theory of cointegration models) with theory of neural networks into one complex system. Moreover, our hybrid neural network implements other than standad learning algorithm in order to make even more accurate predictions.

8. Conclusion

The main goal of this work was to create a new hybrid model of neural network combined with other methods and models (statistical models, machine learning models) which would have better prediction qualities than standard neural network in financial time series forecasting and would hance be a contribution for the area of financial forecasting. We realized this goal by implementing and suggesting a new hybrid model based on neural network and principle of Error-Corretion mechanism coming from the theory of cointegration. Theoretical model combines these two approaches in order to make more accurate predictions. The practical realization of ECM NN model was done by implementing RBF-SEMA computational model.

Except for this, on base on analysis of current state of financial forecasting we suggest also other hybrid models. We theoretically suggested hybrid models RBF-KM, RBF-GA, ARCH-RBF and its recurrent and weighted versions.

In empirical part of the thesis we tested the prediction contribution of ECM NN model as well as prediction contribution of RBF-KM and RBF-GA. We realized experiments on two time series - exchange rates of AUD/USD and EUR/USD. In order to make our results confident, we divided observation into 2 parts - training and validation set. We evaluated the experiments according to methodology stated in Heider et al. (2010).

On base of our experiments one can state that models RBF-KM and RBF-GA provided better prediction qualities than the standard neural network. The main contribution of this work was the creation of new hybrid model - Error-Correction model of neural network. New theory of ECM NN was created, then the model was transformed into computational form and was practically verified on real data. This ECM NN provided significantly better prediction qualities than standard neural network. Besided improved predictions, other advantages of this model are consistency of predictions, nonlinearity in error improvement, consistency of outputs independent of network infrastructure, etc.

Hybrid modelling is a new approach in modelling time series which can produce models with better prediction qualities than standard models. Our ECM NN model showed to be a great contribution (as for prediction qualities) and therefore we believe that this model has a great potential in the domain of financial time series forecasting.

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References

[1] ADYA, M., COLLOPY, F. (1998). *How effective are neural networks at forecasting and prediction? A review and evaluation,* Journal of Forecasting, 17, s. 481 - 495.

[2] AHMEDA, A. F., ATIYAB, N., GAYARC, N. E., EL-SHISHINYD H. (2010). *An Empirical Comparison of Machine Learning Models for Time Series Forecasting,* Econometric Reviews, 29(5), s. 594 - 621.

[3] BÁBEL, J. (2011). *Porovnanie umelých neurónových sietí s modelmi ARCH-GARCH na ekonomických časových radoch,* dizertačná práca, Fakulta riadenia a informatiky, ŽU.

[4] BOX, G. E. P., JENKINS, G. M. (1976). *Time series analysis: forecasting and control,* San Fransisco, CA:Holden-Day.

[5] BOYACIOGLU, M.A, YAKUP, K., OMER, K. B. (2009). Predicting Bank Financial Failures Using Neural Networks, Support Vector Machines and Multivariate Statistical Methods: A Comparative Analysis in the Sample of Savings Deposit Insurance Fund (SDIF) Transferred Banks in Turkey, Expert Systems with Applications, 36, s. 3355 - 3366.

[6] BUHMAN, M. D. (2003). *Radial Basis Functions: Theory and Implementations* Cambridge University.

[7] CLEMEN, R. T. (1989). *Combining forecasts: A review and annotated bibliography. International,* Journal of Forecasting, 5 (1989), 559-583.

[8] CYBENKO, G. (1989). *Approximation by superpositions of a sigmoidal function. Mathematics of Control,* Signals and Systems (MCSS). 2(4), s. 303-314.

[9] DARBELLAY, G. A., & SLAMA, M. (2000). *Forecasting the short-term demand for electricity: Do neural networks stand a better chance?* International Journal of Forecasting, 16, 71- 83.

[10] DHARMISTHA, M. (2012). *Genetic Algorithm based Weights Optimization of Artificial Neural Network,* International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering 1(3).

[11] DAVIS, L. (1987). *Genetic Algorithms and Simulated Annealing,* Pitman, London 1987.

[12] DHAR, V., CHOU, D. (2001). A Comparison of Nonlinear Methods For Predicting Earnings Surprises and Returns, IEEE Transactions on Neural Networks, 12, s. 907 - 921. 12, s. 907 - 921.

[13] DUNIS, CH.. (2007). *Quantitative Trading of Gold and Silver Using Nonlinear Models, Neural Network World,* Vol 16(2), s. 93 - 111.

[14] ENGLE, ROBERT F. (1987). GRANGER, CLIVE W. J. (1987). Co-integration and error correction: Representation, estimation and testing, Econometrica 55 (2): 251-276.

[15] ENGLE, R.F. (1982). *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation,* Econometrica, Vol. 50, No. 4, s. 987 - 1008.

[16] FERNANDEZ, P., TEIXEIRA, J., FERREIRA, J. M., AZEVEDO, S. G. (2008). Modelling Tourism Demand: A Comparative Study Between Artificial Neural Networks and the Box-Jenkins Methodology, Journal For Economic Forecasting, Vol. 5(3), pp. 30 - 50.

[17] GOOIJER, J.G. DE HYNDMAN, R.J. (2006). *25 years of time series forecasting,* Int. J. of Forecasting, 22, 443-473.

[18] HEIDER, D., VERHEYEN, J., HOFFMANN, D. *(2010). Predicting Bevirimat Resistance of HIV-1 from genotype, BMC Bioinformatics 2010, 11:37. .*

[19] HILL, T. L., MARQUEZ, M., OCONNOR, REMUS, W. (1994). *Neural networks models for forecasting and decision making,* Neural Networks, 6, pp. 1069-1072.

[20] HORNIK, K. (1993). *Some new results on neural network approximation,* Neural Networks, 6, pp. 1069-1072.

[21] CHIKHI, M., PÉGUIN-FEISSOLLE, A., TERRAZA, M. (2013). *SEMIFARMA-HYGARCH Modeling of Dow Jones Return Persistence,*. Computational Economics, 41, 249-265.

[22] CHOUDHRY, R., GARG K. (2008). *A Hybrid Machine Learning System for Stock Market Forecasting,* World Academy of Science, Engineering and Technology 39.

[23] MACIEL, L. S., BALLINI, R. (2008): *Design a Neural Network for Time Series Financial Forecasting: Accuracy and Robustness Analysis,* 2008.

[24] MALLIARIS, M., MALLIARIS, A. G. (2009). Modeling Federal Funds Rates: A Comparison of Four Methodologies, Neural Computing and Applications, 18, s. 37 - 44.

[25] MARČEK, D., MARČEK, M. (2006). *Neurónové siete a ich aplikácie,* Žilina: EDIS - Vydavateľstvo ŽU, 223p.

[26] MARČEK, D., MARČEK, M., PANČÍKOVÁ, L. (2008). *Ekonometria a soft computing,* EDIS - Vydavateľstvo ŽU, Žilina.

[27] MONTGOMMERY, D.C., JENNINGS, C.L., KULAHCI, M. (2008). *Introduction to Time Series Analysis and Forecasting,* , New Jersey: John Wiley & Sons, Inc, 445p. ISBN 978-0-471-65397-4.

[28] ORR, M. J. L. (1996). *Introduction to Radial Basis Function Networks, Technical Report April, Center for Cognitive Science,* University of Edinburgh.

[29] O'DONOVAN, T. M., (1983). *Short Term Forecasting: An Introduction to the Box-Jenkins Approach.* NY: New York, Wiley.

[30] PARK, C. H. O., IRWIN, S. H. (2004). *The profitability of technical analysis: A review.* Champaign: Urbana.

[31] QI, M. (2001). *Predicting US recessions with leading indicators via neural network models.* International Journal of Forecasting, 17, s. 383- 401.

[32] SADORSKY, P. (2006). *Modeling and forecasting petroleum futures volatility,* Energy Economics, 28(4) (2006) s. 467-488.

[33] ŠTERBA, J., HIĽOVSKA, K. (2010): *The Implementation of Hybrid ARIMA-Neural Network Prediction Model for Agregate Water Consumption Prediction,* Journal of Applied Mathematics, Volume 3 (2010), Number 3.

[34] THINYANE, H., MILLIN, J. (2011). *An Investigation into the Use of Intelligent Systems for Currency Trading,* Computational Economics, 37, s. 363-374.

[35] TRUCK, S., LIANG, K. (2012). *Modelling and Forecasting Volatility in the Gold Market,* International Journal of Banking and Finance, 9(1) (2012) 47 - 78.

[36] WU, Q., HAO, G. (2009). *Forecasting Educational Expenditure Based on Radial Basis Function Neural Network and Principal Component Analysis,* Proceedings of 2009 WRI: World Congress on Software Engineering - Vol. 4 IEEE Computer Society, Washington DC.

[37] YANG, Y. (2000). *Combining Forecasting Procedures: Some Theoretical Results,* Econometric Theory, 20, s. 176-222.

[38] YAO, J., TAN, C. L. (2000). A Case Study On Using Neural Networks to Perform Technical Forecasting on Forex, Neurocomputing, 34, s. 79 - 98.

[39] YU, L., SHOUYANG, W., KIM, K. L. (2007). *Foreign Exchange Rate Forecasting with Artificial Neural Networks,* Springer, ISBN: 0387717196.

[40] ZHANG, G. P., QI, M. (2002). *Predicting consumer retail sales using neural networks. In Smith and Gupta, Neural Networks in Business,* Techniques and Applications, Hershey PA: Idea Group Publishing, s. 26 - 40.

[41] ZHOU, G. CH. (2011). Solution to Chinas GDP Prediction Problem by BP Neural Network, Applied Mechanics and Materials, 50-51.

[42] ZHUANG, X.-F., CHAN, L.-W. (2004). *Volatility Forecasts in Financial Time Series with HMM-GARCH Models,* IDEAL 2004, LNCS 3177, pp. 807-812, Springer-Verlag Berlin Heidelberg 2004.

Selected Papers by the Author

1. FALÁT, L., MARČEK, D. (2014). Financial Time Series Modelling with Hybrid Model Based on Customized RBF Neural Network Combined With Genetic Algorithm, Advances in Electrical and Electronic Engineering, Vol. 12, No. 4 (2014), s. 307 - 318.

2. FALÁT, L., MARČEK, D. (2014). Volatility Forecasting in Financial Risk Management with Statistical Models and ARCH-RBF Neural Networks, Journal of Risk Analysis and Crisis Response, Vol. 4, Issue 2 (2014), s. 77 - 95.

3. FALÁT, L., MARČEK, D. (2014). *Forecasting AUD/USD Exchange Rate with RBF Neural Network Combined with Evolutionary Approach and Unsupervised Learning* Zborník z konferencie IT for Practice 2014, s. 57 - 66.

4. FALÁT, L., MARČEK, D. (2014). *(G)ARCH-RBF Modelling of Conditional Volatility in Finance,* konferencia Strategic Management and its Support by Information Systems 2015.

5. FALÁT, L., MARČEK, D. (2014). In *Forecasting Financial Volatility: Application of Customized Neural Network Combined with ARCH Model into Time Series Modelling,* Zborník z konferencie Letní škola informatiky Masarykovy univerzity 2014.

6. FALÁT, L., KOTTILOVÁ, A. (2012). Forecasting on Forex Market with RBF and Perceptron Neural Networks, AD ALTA: J. of Interdisciplinary Research (Double-Blind Peer-Reviewed)

7. FALÁT, L., TOKARČÍKOVÁ, E. (2013). Application of Soft Computing Techniques into the Decision-Making Process of Economical Subjects, Proc. of Int. Masaryk Conf. for PhD Students and Young Researchers, Magnamitas, Hradec Králové, Česká republika, 2013, s. 705 - 713.

8. MARČEK, D., FALÁT, L., (2013) Forecasting of Financial Time Series Using Artificial Neural Networks with Learning Based on Evolutionary Approach and Clustering, Proc. of Int. Masaryk Conf. for PhD Students and Young Researchers, Magnamitas, Hradec Králové, Česká republika, 2013, s. 1485 - 1491.

9. FALÁT, L., PANČÍKOVÁ, L. (2012). *Introduction to Statistical Modelling in Gretl and its Application in Econometrics,* J. of Information and Control and Management Systems, Vol. 10, No.2 (2012).

10. FALÁT, L. (2012). *Artificial Neural Networks Improvement: Hybridization with Statistical Models,* Časopis Slovenskej spoločnosti pre systémovú integráciu, No 1 (2012), ISSN 1336-591.

11. FALÁT, L. (2012). *Financial Forecasting Using Artificial Neural Networks,* Zborník z konferencie, 5th Int. Scientific-technical Conf.: Infocommunicational Technologies in Science, Production and Education (Infocom 5), Ruská federácia, ISSN 2219-293X, s. 155 - s. 160.

12. KOTILLOVÁ A., FALÁT, L. (2012). *RBF ARIMA Hybrid Model for Electricity Demand Prediction,* Zborník 12. mezinárodní konference IMEA 2012, Hradec Králové, 10.-11.máj 2012.

13. FALÁT, L., KOTILLOVÁ, A. (2012). Artificial Neural Networks Modelling: Application to Exchange Rates Data, Zborník interdisciplinárnej medzinárodnej vedeckej konferencie doktorandov a odborných asistentov QUAERE 2012, ISBN 978-80-905243-0-9, s. 925 - 931.

14. FALÁT, L. (2012). *Data Analysis and Modeling Using Open Source Software R,* Proceedings of Tenth International Conference on Soft Computing Applied in Computer and Economic Environments - ICSC 2012, ISBN 978-80-7314-279-7, s.37 - s.42.

15. FALÁT, L. (2012). *Numerical Analysis of Learning Algorithms and Strategies of Multilayer Perceptron Neural Network,* Proceedings in Information and Communication Technologies - International Conference, 2012, ISBN 978-80-554-0513-1, s.12 - 17.

16. FALÁT, L. (2012). Prediction Models as a Support in Managerial Decision Making Process, RecenzovanÃ¡ Zborník interdisciplinárnej medzinárodnej vedeckej konferencie doktorandov a odborných asistentov QUAERE 2012, ISBN 978-80-905243-0-9, s. 124 - s. 132.

17. FALÁT, L., HORVÁTH, M., MARČEK, D. (2011). *Computer Processing of Portfolios and Investment on Portfolios,* Zborník Jak úspěšně podnikat v příhraničních regionech jihovýchodní Moravy, VI. mezinárodní konference, Evropský polytechnický institute, ISBN 978-80-7314-238-4, s.75 - s.81.

18. FALÁT, L. (2011). *Hybrid Neural Networks as a New Approach in Time Series Forecasting,* AD ALTA: Journal of Interdisciplinary Research (Double-Blind Peer-Reviewed), Volume 2, 2011, ISSN 1804-7890.